Objective: This question deals with Series Solution of DEs

A) Find a power series solution in powers of x to the following DE:

\[ y'' - y = 0 \]

Hint:

\[ e^x = \sum_{m=0}^{\infty} \frac{x^m}{m!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \]
\[ \cosh x = \sum_{m=0}^{\infty} \frac{x^{2m}}{(2m)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots \]
\[ \cos x = \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m)!} x^{2m} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \]
\[ \sinh x = \sum_{m=0}^{\infty} \frac{x^{2m+1}}{(2m+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \]
\[ \sin x = \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)!} x^{2m+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \]

B) Express a general solution to the following equation using Bessel’s equation of the first kind

\[ 4x^2 y'' + 4xy' + (4x^2 - 1)y = 0 \]

Objective: This question deals with first order DE.

a) Test for exactness. If exact, solve. If not find the IF, then solve it

\[ (e^{x+y} - y)dx + (xe^{x+y} + 1)dy = 0 \]