Course: Biostatistics

Lecture No: [2]

Chapter: [1]

Introduction to Statistics

Section: [1.1]

An Overview of Statistics

What is DATA?

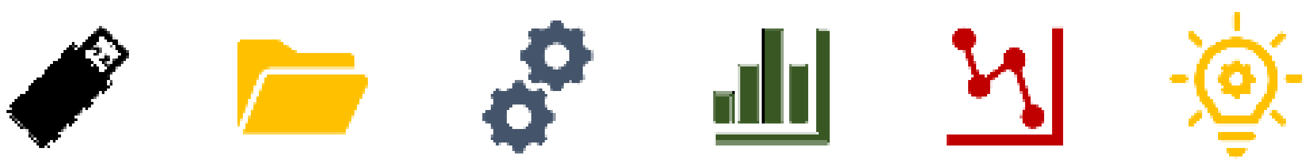
Data: Consist of *information* coming from <u>observations</u>, <u>counts</u>, <u>measurements</u>, or <u>responses</u>.

Example: According to a survey, more than 7 in 10 Americans say a nursing career is a prestigious occupation (مهنة مرموقة).

Example: "Social media consumes kids today as well, as more score their first social media accounts at an average age of 11.4 years old."

What is STATISTICS?

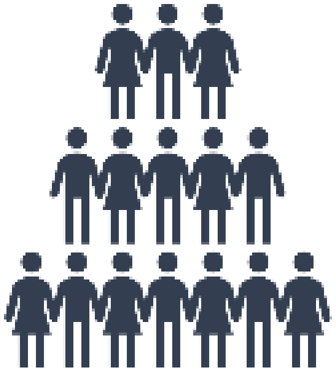
Statistics: The science of collecting, organizing, analyzing, and interpreting data in order to make decisions.



Data Sets

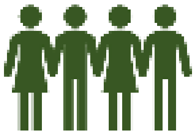
Population:

The collection of ***all*** outcomes, responses, measurements, or counts that are of interest.



Sample:

A **subset**, or part, of the *population*.



Data Sets

Example: Identifying Data Sets

In a recent survey, 834 employees in the United States were asked if they thought their jobs were highly stressful. Of the 834 respondents, 517 said yes.

1. **Identify the population and the sample.**

   Population: the responses of all employees in the U.S.

   Sample: the responses of the 834 employees in the survey.

2. **Describe the sample data set.**

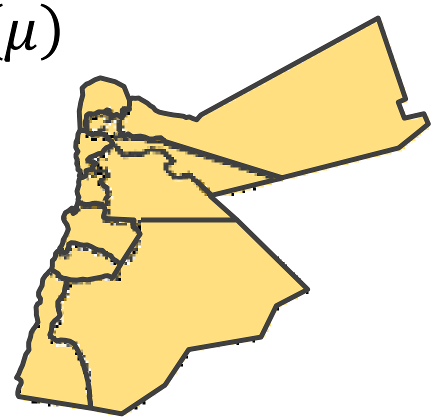
   The data set consists of 517 **YES**'s and 317 **NO**'s.

Parameter and Statistic

Parameter

A numerical description of a population characteristic.

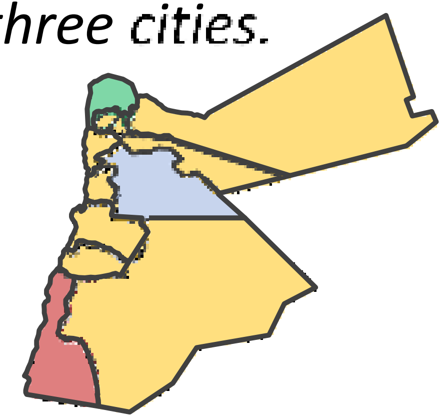
Average age of all people in JORDAN. (μ)



Statistic

A numerical description of a sample characteristic.

Average age of people from a sample of three cities.
($\bar{x}$)



Parameter and Statistic

Example: Decide whether each number describes a **population parameter** or a **sample statistic**.

1 A survey of several hundred collegiate student-athletes in the United States found that, during the season of their sport, the average time spent on athletics by student-athletes is 50 hours per week.

Because the average of 50 hours per week is based on a subset of the population, it is a sample statistic.

Parameter and Statistic

Example: Decide whether each number describes a **population parameter** or a **sample statistic**.

2 The freshman class at a university has an average SAT math score of 514.

Because the average SAT math score of 514 is based on the entire freshman class, it is a population parameter.

Parameter and Statistic

Example: Decide whether each number describes a **population parameter** or a **sample statistic**.

3 In a random check of several hundred retail stores, the Food and Drug Administration found that 34% of the stores were not storing fish at the proper temperature.

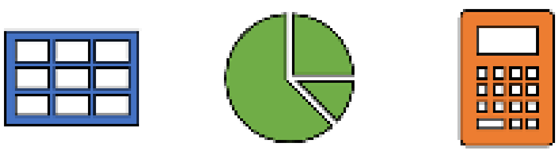
Because 34% is based on a subset of the population, it is a sample statistic.

Branches of Statistics

Descriptive Statistics

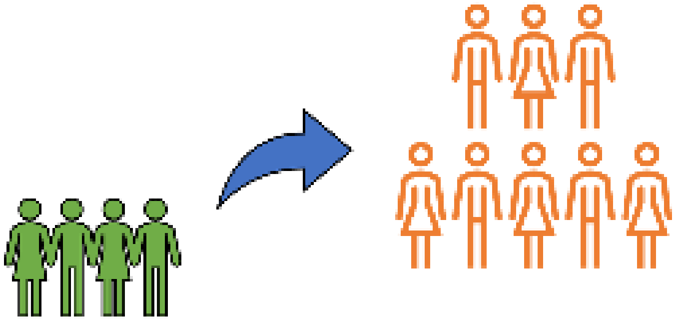
Involves organizing, summarizing, and displaying data.

Tables, charts, averages.



Inferential Statistics

Involves using *sample data* **to draw conclusions about a population.**



Branches of Statistics

Example:

For the following study:

1. Identify the population and the sample.
2. Then determine which part of the study represents the descriptive branch of statistics.
3. What conclusions might be drawn from the study using inferential statistics?

A study of 2560 U.S. adults found that of adults not using the Internet, 23% are from households earning less than $30000 annually, as shown in the figure.

[1] The population consists of the responses of all U.S. adults, and the sample consists of the responses of the 2560 U.S. adults in the study.

Branches of Statistics

Example:

For the following study :

1. Identify the population and the sample.
2. Then determine which part of the study represents the descriptive branch of statistics.
3. What conclusions might be drawn from the study using inferential statistics?

A study of 2560 U.S. adults found that of adults not using the Internet, 23% are from households earning less than $30000 annually, as shown in the figure.

[2] The descriptive branch of statistics involves the statement 23% of U.S. adults not using the Internet are from households earning less than $30000 annually.

Branches of Statistics

Example:

For the following study :

1. Identify the population and the sample.
2. Then determine which part of the study represents the descriptive branch of statistics.
3. What conclusions might be drawn from the study using inferential statistics?

A study of 2560 U.S. adults found that of adults not using the Internet, 23% are from households earning less than $30000 annually, as shown in the figure.

[3] A possible inference drawn from the study is that lower-income households cannot afford access to the Internet.

Couse: Biostatistics

Lecture No: [2]

Chapter: [1]

Introduction to Statistics

Section: [1.2]

Data Classification

Types of Data

Qualitative Data

Consists of attributes, labels, or nonnumerical entries.

- *Major*
- *Place of Birth*
- *Eye color*

Quantitative Data

Numerical measurements or counts.

- *Age*
- *Weight*
- *Temperature*

Course: Biostatistics Lecture No: [3]

Chapter: [1]
Introduction to Statistics

Section: [1.3]
Data Collection and Experimental Design

DESIGN OF A STATISTICAL STUDY

- The *goal* of every statistical study is to collect data and then use the data to make a decision.
- Before interpreting the results of a study, you should be familiar with *how to design a statistical study*.

DESIGN OF A STATISTICAL STUDY

Designing a Statistical Study

1. Identify the *variable(s) of interest* (the focus) and the *population of the study*.
2. Develop a detailed *plan for collecting data*. If you use a sample, make sure the sample is *representative of the population*.
3. Collect the data.
4. Describe the data, using descriptive statistics techniques.
5. Interpret the data and *make decisions about the population using inferential statistics.*
6. Identify any possible errors.

DESIGN OF A STATISTICAL STUDY

Categories of a Statistical Study

Observational Study

A researcher does not influence the responses.

Example: an observational study was performed in which researchers observed and recorded the mouthing behavior on nonfood objects of children up to three years old.

Experiment

A researcher deliberately (متعمداً) applies a treatment before observing the responses.

DESIGN OF A STATISTICAL STUDY

Categories of a Statistical Study

Experiment

A researcher deliberately (متعمداً) applies a treatment before observing the responses.

- A **treatment** is applied to part of a population, called a *treatment group*, and responses are observed.
- *Another part* of the population may be used as a *control group*, in which **no treatment is applied**. (The subjects in both groups are called *experimental units*.)
- In many cases, subjects in the **control group** are given a *placebo*, which is a harmless, fake treatment that is made to look like the real treatment.

DESIGN OF A STATISTICAL STUDY

Categories of a Statistical Study

Experiment

A researcher deliberately (متعمداً) *applies a treatment before observing the responses*.

- It is a good idea to use the same number of subjects for each group.

Example:

An experiment was performed in which diabetics took cinnamon extract daily while a control group took none. After 40 days, the diabetics who took the cinnamon reduced their risk of heart disease while the control group experienced no change.

DESIGN OF A STATISTICAL STUDY

Example: Observational Study or an Experiment ?

1. Researchers study the effect of vitamin D_3 supplementation among patients with antibody deficiency or frequent respiratory tract infections. To perform the study, 70 patients receive 4000 IU of vitamin D_3 daily for a year. Another group of 70 patients receive a placebo daily for one year.

Because the study applies a treatment (vitamin D3) to the subjects, the study is an experiment.

DESIGN OF A STATISTICAL STUDY

Example: Observational Study or an Experiment ?

2 Researchers conduct a study to find the U.S. public approval rating of the U.S. president. To perform the study, researchers call 1500 U.S. residents and ask them whether they approve or disapprove of the job being done by the president.

Because the study does not attempt to influence the responses of the subjects (there is no treatment), the study is an observational study.

DATA COLLECTION

Simulation

- Uses a mathematical or physical model to reproduce the conditions of a situation or process.
- Often involves the use of computers.
- Allow you to study situations that are impractical or even dangerous to create in real life.
- Often save time and money.
- **Example:** automobile manufacturers use simulations with dummies to study the effects of crashes on humans.

Survey

- An investigation of one or more characteristics of a population.
- Surveys are carried out on people by asking them questions.
- Commonly done by interview, Internet, phone, or mail.
- In designing a survey, it is important to word the questions so that they do not lead to biased results, which are not representative of a population.

EXPERIMENTAL DESIGN

- To produce meaningful unbiased results, experiments should be carefully designed and executed.
- Three key elements of a well-designed experiment are:

1. Control
2. Randomization
3. Replication

EXPERIMENTAL DESIGN **Control**

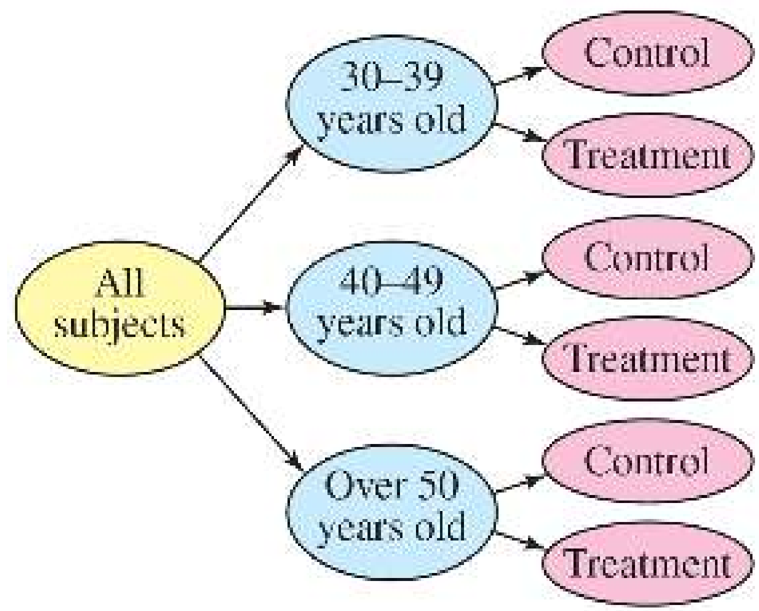
- Because experimental results can be ruined (تتأثر) by a variety of factors, being able to control these influential factors (العوامل المؤثرة) is important. One such factor is a *confounding variable* (المتغير المضلل).
- A **confounding variable** occurs when an experimenter cannot tell the difference between the effects of different factors on the variable.
- **Example:** A coffee shop owner remodels her shop at the same time a nearby mall has its grand opening. If business at the coffee shop increases, it cannot be determined whether it is because of the remodeling or the new mall.

EXPERIMENTAL DESIGN Control

- Another factor that can affect experimental results is the placebo effect.
- The *placebo effect* occurs when a subject reacts favorably to a placebo when in fact the subject has been given a fake treatment.
- To help control or minimize the placebo effect, a technique called blinding can be used.
- **Blinding** is a technique where the subject does not know whether he or she is receiving a treatment or a placebo.
- **Double-Blind** experiment neither the subject nor the experimenter knows if the subject is receiving a treatment or a placebo.

EXPERIMENTAL DESIGN Randomization

- **Randomization** is a process of randomly assigning subjects to different treatment groups.
- **Randomized block design**: Divide subjects with similar characteristics into blocks, and then within each block, randomly assign subjects to treatment groups.
- **Example:** An experimenter who is testing the effects of a new weight loss drink may first divide the subjects into age categories and then, within each age group, randomly assign subjects to either the treatment group or the control group



EXPERIMENTAL DESIGN Replication

- **Replication** is the repetition of an experiment under the same or similar conditions.
- **Sample size**, which is the number of subjects in a study, is another important part of experimental design.
- **Example:** suppose an experiment is designed to test a vaccine against a strain of influenza. In the experiment, 10,000 people are given the vaccine and another 10,000 people are given a placebo. Because of the sample size, the effectiveness of the vaccine would most likely be observed. But, if the subjects in the experiment are not selected so that the two groups are similar (according to age and gender), the results are of less value.

EXPERIMENTAL DESIGN

Example A company wants to test the effectiveness of a new gum developed to help people quit smoking. Identify a potential problem with the given experimental design and suggest a way to improve it.

[1] The company identifies ten adults who are heavy smokers. Five of the subjects are given the new gum and the other five subjects are given a placebo. After two months, the subjects are evaluated, and it is found that the five subjects using the new gum have quit smoking.

Problem

The sample size being used is not large enough.

Solution

The experiment must be replicated

EXPERIMENTAL DESIGN

Example A company wants to test the effectiveness of a new gum developed to help people quit smoking. Identify a potential problem with the given experimental design and suggest a way to improve it.

[2] The company identifies one thousand adults who are heavy smokers. The subjects are divided into blocks according to gender. After two months, the female group has a significant number of subjects who have quit smoking.

Problem

The groups are not similar. The new gum may have a greater effect on women than men, or vice versa.

Solution

They must be randomly assigned to be in the treatment group or the control group.

Sampling Techniques

- A **census** (التعداد السكاني) is a count or measure of an *entire* population.
- Taking a census provides complete information, but it is often *costly and difficult to perform*.
- A **sampling** is a count or measure of part of a population and is more commonly used in statistical studies.
- To collect *unbiased* (غير متحيز) data, a researcher must ensure that the sample is *representative of the population*.
- Even with the best methods of sampling, a *sampling error* may occur.
- A **sampling error** is the difference between the results of a sample and those of the population.

Sampling Techniques

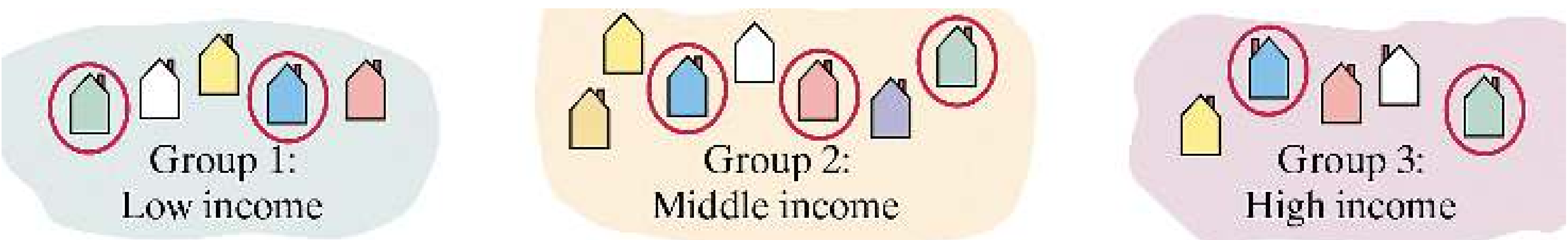
- A **random sample** is one in which every member of the population has an *equal chance of being selected*.
- A **simple random sample** is a sample in which every possible sample of the same size has the same chance of being selected.
- One way *to collect* a simple random sample is to assign a different number to each member of the population and then use a random number tables, calculators or computer software programs to generate random numbers.

Sampling Techniques

- When you choose members of a sample, you should decide whether it is acceptable to have the *same population member selected more than once.*
- If it is *acceptable*, then the sampling process is said to be with replacement. If it is *not acceptable*, then the sampling process is said to be without replacement.
- There are several other commonly used sampling techniques. Each has advantages and disadvantages.
  1. Stratified Sample (العينة الطبقية)
  2. Cluster Sample (العينة العنقودية)
  3. Systematic Sample (العينة المنهجية)
  4. Convenience Sample (العينة المريحة)

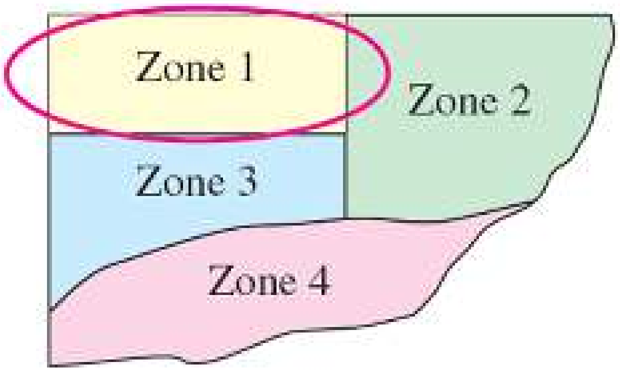
Sampling Techniques   Stratified Sample

- Divide a population into groups (strata) and select a random sample from each group.
- **Example:** To collect a stratified sample of the number of people who live in Amman households, you could divide the households into socioeconomic (الوضع الاقتصادي والاجتماعي) levels and then randomly select households from each level.



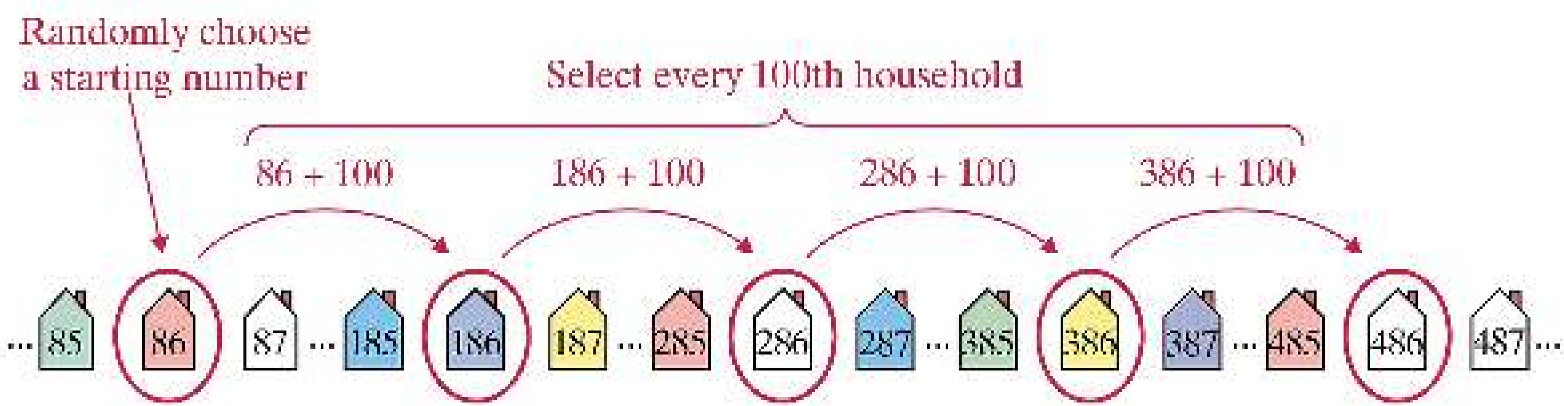
Sampling Techniques **Cluster Sample**

- Divide the population into groups (clusters) and select all of the members in one or more, but not all, of the clusters.
- In the Amman example you could divide the households into clusters according to zones, then select all the households in one or more, but not all, zones.



Sampling Techniques Systematic Sample

- Choose a starting value at random. Then choose every k^{th} member of the population.
- In the Amman example you could assign a different number to each household, randomly choose a starting number, then select every 100^{th} household.



Sampling Techniques   Convenience Sample

- Choose only members of the population that are easy to get.
- Often leads to biased studies (**not recommended**).

Sampling Techniques

Example: *Identifying Sampling Techniques.*

You are doing a study to determine the opinion of students at your school regarding stem cell research. Identify the sampling technique used.

[1] You divide the student population with respect to majors and randomly select and question some students in each major.	[2] You assign each student a number and generate random numbers. You then question each student whose number is randomly selected.	[3] You select students who are in your biology class.
Stratified Sampling	**Simple Random Sample**	**Convenience Sample**

Course: Biostatistics

Lecture No: [4]

Chapter: [2]

Descriptive Statistics

Section: [2.3]

Measures of Central Tendency

Where You have Been

- You learned that there are many ways to collect data.
- Usually, researchers must work with sample data in order to analyze populations.
- Occasionally (بين الحين والآخر), it is possible to collect all the data for a given population.

Where You are Going

- You will take a review of some ways to organize and describe data sets.
- The goal is to make the data easier to understand by describing trends, averages, and variations.

MEAN

- A measure of central tendency is a value that represents a typical, or central, entry of a data set.
- The three most used measures of central tendency are
    1. the *mean*,
    2. the *median*,
    3. and the *mode*.

MEAN

- The **mean** (*average*) of a data set is the sum of the data entries ($\sum x$) divided by the number of entries (n or N).
- To find the mean of a data set, use one of these formulas:

✓ Population Mean: $\mu = \frac{\sum x}{N}$

✓ Sample Mean: $\bar{x} = \frac{\sum x}{n}$

MEAN

Example: The weights (in pounds) for a sample of adults before starting a weight-loss study are listed. What is the mean weight of the adults?

274 235 223 268 290 285 235

$$\bar{x} = \frac{\sum x}{n}$$
$$= \frac{274 + 235 + 223 + 268 + 290 + 285 + 235}{7}$$
$$= \frac{1810}{7} \approx 258.57$$

The mean weight of the adults is about 258.6 pounds

MEAN

- **Advantage of using the mean:**
  - ✓ The mean is a reliable measure because it considers every entry of a data set.
- **Disadvantage of using the mean:**
  - ✓ Greatly affected by *outliers* (a data entry that is far removed from the other entries in the data set).

MEAN OF GROUPED DATA [WEIGHTED MEAN]

Example: Consider the following data:

1	4	3	2	3
2	2	1	3	4
3	1	3	1	1
2	4	1	2	3

$$\bar{x} = \frac{1 + 4 + 3 + \cdots + 1 + 2 + 3}{20}$$?!

x	Frequency (f)
1	6
2	5
3	6
4	3
	20

$$\bar{x} = \frac{\sum(x \cdot f)}{\sum f}$$

$$= \frac{(1 \times 6) + (2 \times 5) + (3 \times 6) + (4 \times 3)}{20}$$

$$= \frac{6 + 10 + 18 + 12}{20}$$

$$= 2.3$$

Course: Biostatistics

Lecture No: [5]

Chapter: [2]

Descriptive Statistics

Section: [2.4]

Measures of Variation

RANGE

- In this section, you will learn different ways to measure the variation (or *spread*) of a data set.
- The simplest measure is the range of the set.
- Range = (*Maximum* data entry) − (*Minimum* data entry)
- The range has the *advantage* of being easy to compute.
- Its *disadvantage* is that it uses only two entries from the data set.

VARIANCE AND STANDARD DEVIATION

- Two measures of variation that use all the entries in a data set.
- Before you learn about these measures of variation, you need to know what is meant by the *deviation* of an entry in a data set.
- The **deviation** of an entry x in a population data set is the difference between the entry and the mean μ of the data set.
- [Deviation of x] $= x - \mu$
- $\sum(x - \mu) = 0$

VARIANCE AND STANDARD DEVIATION

Example:

x	$x - \mu$
11	3
7	-1
3	-5
13	5
6	-2

$\Sigma = 0$

The average of the data is $\mu = 8$.

VARIANCE AND STANDARD DEVIATION

Population Variance

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N}$$

$$= \frac{\sum(x^2) - \frac{(\sum x)^2}{N}}{N}$$

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}} = \sqrt{\frac{\sum(x^2) - \frac{(\sum x)^2}{N}}{N}}$$

VARIANCE AND STANDARD DEVIATION

Example: Find the population variance of the following data.

$N = 8$

$$\mu = \frac{14 + 12 + 6 + 13 + 2 + 11 + 9 + 5}{8}$$
$$= 9$$

x	$(x - \mu)^2$
14	25
12	9
6	9
13	16
2	49
11	4
9	0
5	16
	128

Variance $\sigma^2 = \frac{128}{8} = 16$

Standard Deviation: $\sigma = \sqrt{16} = 4$

VARIANCE AND STANDARD DEVIATION

Sample Variance

$$s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$= \frac{\sum(x^2) - \frac{(\sum x)^2}{n}}{n-1}$$

Population Standard Deviation

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum(x^2) - \frac{(\sum x)^2}{n}}{n-1}}$$

VARIANCE AND STANDARD DEVIATION

Example: Find the sample variance of the following data.

x	x^2
10	100
5	25
3	9
6	36
8	64
2	4
4	16
14	196
52	450

$n = 8$

Variance $$s^2 = \frac{450 - \frac{(52)^2}{8}}{8 - 1} = 16$$

Standard Deviation: $$s = \sqrt{16} = 4$$

VARIANCE AND STANDARD DEVIATION

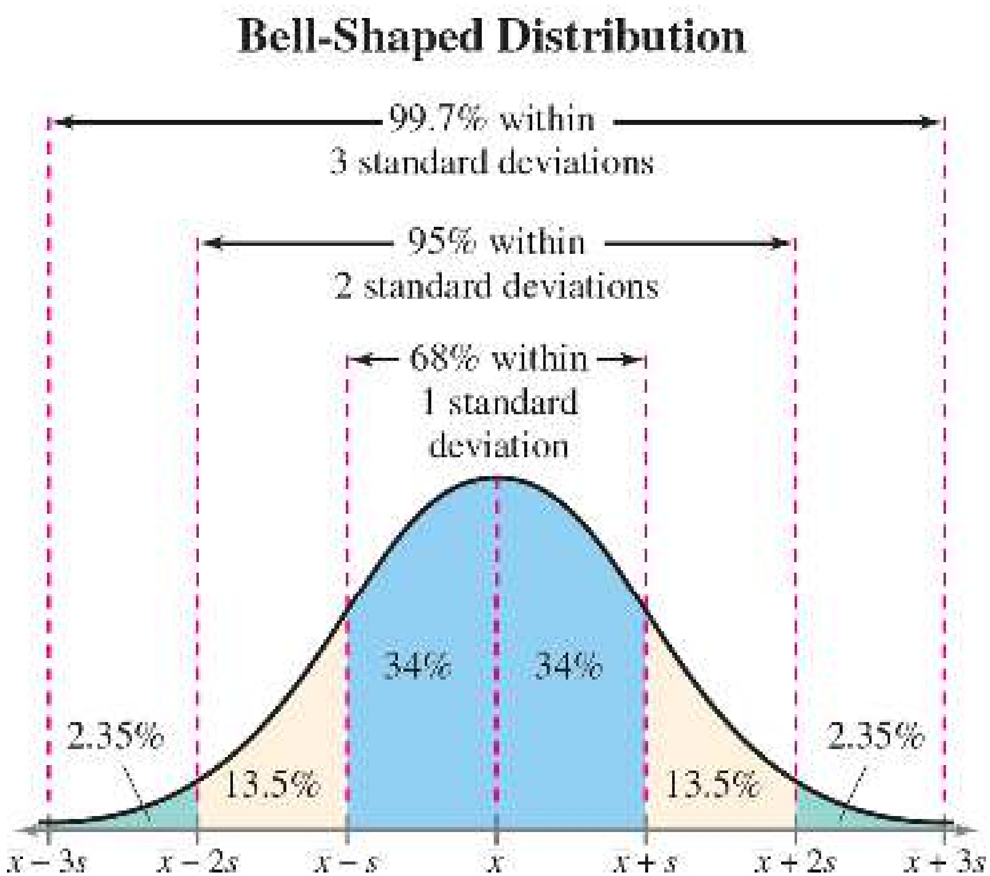
NOTES

- The standard deviation measures the variation of the data set about the mean and has the same units of measure as the data set.
- The standard deviation is always greater than or equal to 0.
- When $\sigma = 0$, the data set has no variation, and all entries have the same value.
- As the entries get farther from the mean (that is, more spread out), the value of σ increases.

EMPIRICAL RULE (or 68 – 95 – 99.7 RULE)

For data sets with distributions that are approximately symmetric and bell-shaped, the standard deviation has these characteristics.

- About 68% of the data lie within one standard deviation of the mean.
- About 95% of the data lie within two standard deviations of the mean.
- About 99.7% of the data lie within three standard deviations of the mean.



Course: Biostatistics

Lecture No: [5]

Chapter: [5]

Normal Probability Distributions

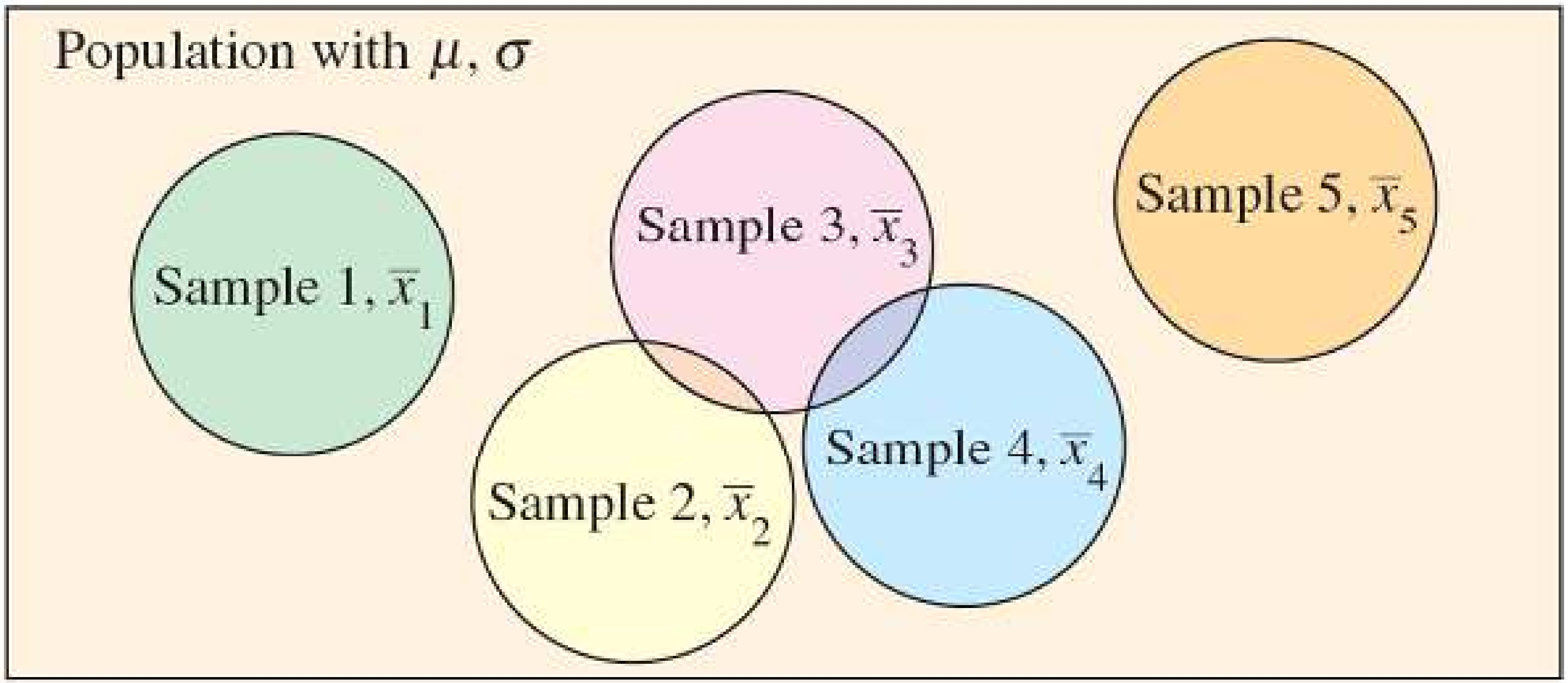
Section: [5.4]

Sampling Distributions and the Central Limit Theorem

SAMPLING DISTRIBUTIONS

- In this section, you will study the *relationship* between a population mean and the means of samples taken from the population.
- A **sampling distribution** is the probability distribution of a *sample statistic* that is formed when samples of size n are repeatedly taken from a population.
- If the sample statistic is the *sample mean*, then the distribution is the sampling distribution of sample means.
- Every sample statistic has a sampling distribution.

SAMPLING DISTRIBUTIONS



PROPERTIES OF SAMPLING DISTRIBUTIONS OF SAMPLE MEANS

- The mean of the sample means $\mu_{\bar{x}}$ is equal to the population mean μ.

$$\mu_{\bar{x}} = \mu$$

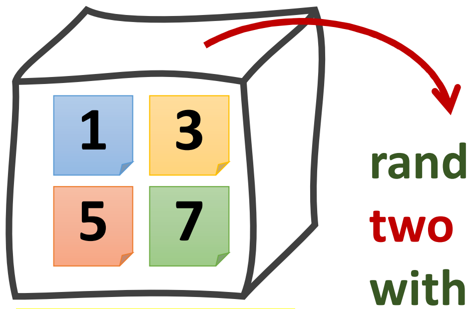
- The standard deviation of the sample means $\sigma_{\bar{x}}$ is equal to the population standard deviation σ divided by the square root of the sample size n.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- The standard deviation of the sampling distribution of the sample means is called the **standard error of the mean**.

A SAMPLING DISTRIBUTION OF SAMPLE MEANS

Example



Population

randomly choose two slips of paper, with replacement.

x	Freq.
1	1
3	1
5	1
7	1

- **Population Mean:**

$$\mu = \frac{1+3+5+7}{4} = 4$$

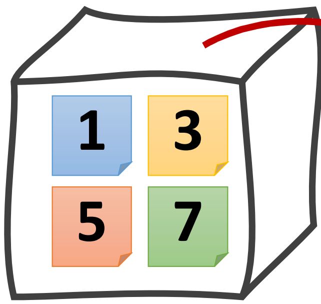
- **Population Standard Deviation:**

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N} = \frac{9+1+1+9}{4} = 5$$

$$\therefore \sigma = \sqrt{5}$$

A SAMPLING DISTRIBUTION OF SAMPLE MEANS

Example



Samples

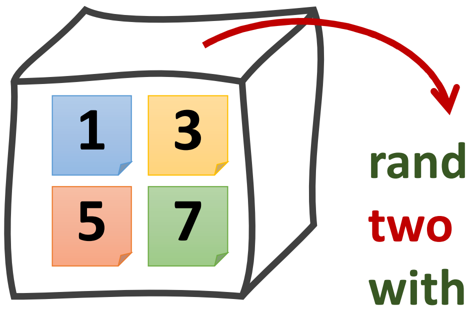
randomly choose **two slips** of paper, with replacement.

$\mu = 4 \quad \sigma = \sqrt{5}$

1	1	1	3	1	2	5	1	3	7	1	4
1	3	2	3	3	3	5	3	4	7	3	5
1	7	4	3	7	5	5	7	6	7	7	7
1	5	3	3	5	4	5	5	5	7	5	6

A SAMPLING DISTRIBUTION OF SAMPLE MEANS

Example



Samples

randomly choose two slips of paper, with replacement.

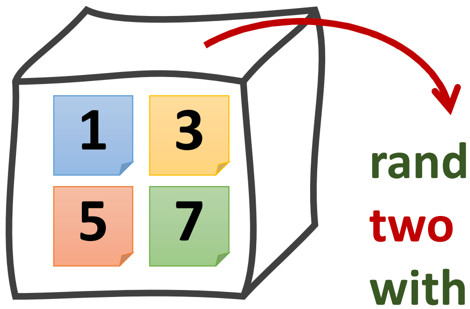
$\mu = 4 \quad \sigma = \sqrt{5}$

1	2	3	4
2	3	4	5
4	5	6	7
3	4	5	6

$\bar{x}$	Freq.
1	1
2	2
3	3
4	4
5	3
6	2
7	1

A SAMPLING DISTRIBUTION OF SAMPLE MEANS

Example



Samples

randomly choose **two slips** of paper, with replacement.

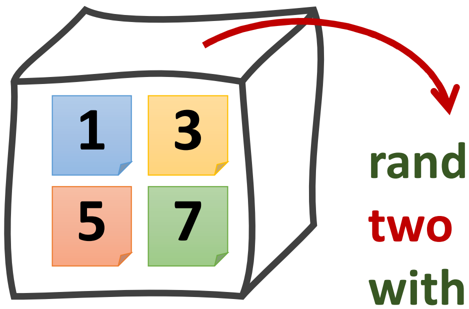
$$\mu = 4 \qquad \sigma = \sqrt{5}$$

$$\mu_{\bar{x}} = \frac{\sum(\bar{x} \cdot f)}{\sum f} = \frac{64}{16} = 4 = \mu$$

$\bar{x}$	Freq.	$\bar{x} \cdot f$
1	1	1
2	2	4
3	3	9
4	4	16
5	3	15
6	2	12
7	1	7
	16	**64**

A SAMPLING DISTRIBUTION OF SAMPLE MEANS

Example



Samples

randomly choose **two slips** of paper, with replacement.

$\mu = 4 \quad \sigma = \sqrt{5}$

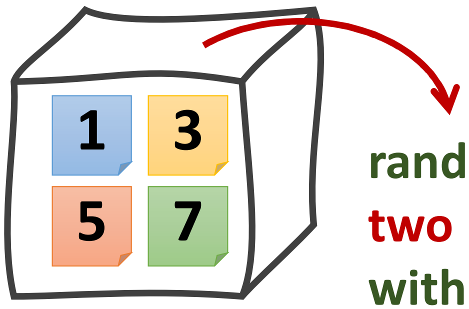
$$\mu_{\bar{x}} = \frac{\sum(\bar{x} \cdot f)}{\sum f} = \frac{64}{16} = 4 = \mu$$

$$\sigma_{\bar{x}}^2 = \frac{\sum[(\bar{x} - \mu_{\bar{x}})^2 \cdot f]}{\sum f} = \frac{40}{16} = \frac{5}{2} = \frac{\sigma^2}{n}$$

$\bar{x}$	Freq.	$\bar{x} \cdot f$	$(\bar{x} - \mu_{\bar{x}})^2 \cdot f$
1	1	1	9
2	2	4	8
3	3	9	3
4	4	16	0
5	3	15	3
6	2	12	8
7	1	7	9
	16	**64**	**40**

A SAMPLING DISTRIBUTION OF SAMPLE MEANS

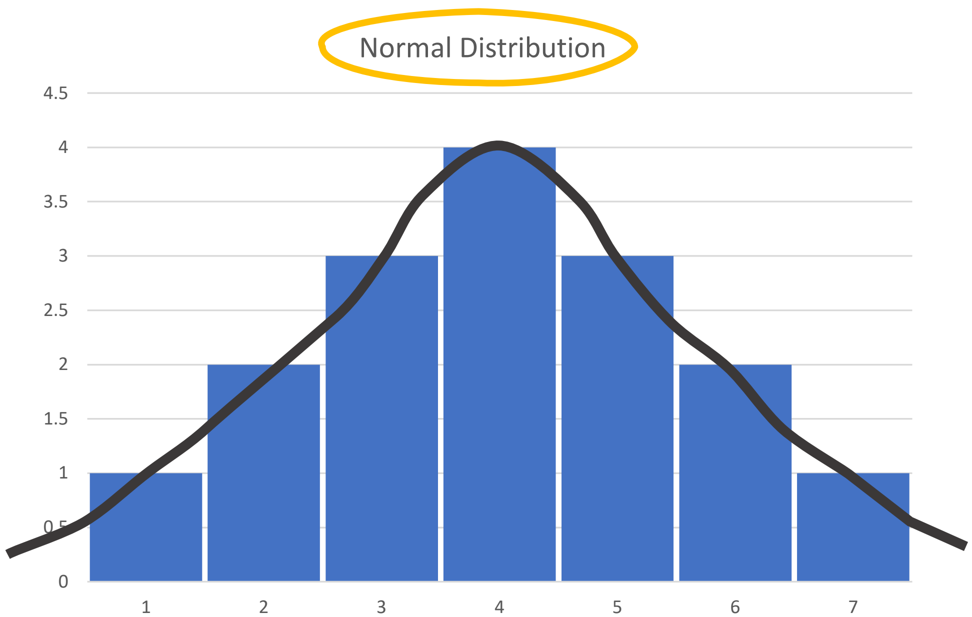
Example



Samples

randomly choose two slips of paper, with replacement.

$\bar{x}$	Freq.
1	1
2	2
3	3
4	4
5	3
6	2
7	1



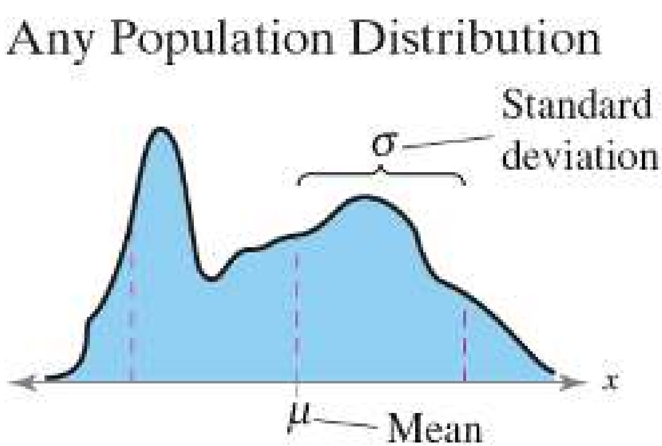
THE CENTRAL LIMIT THEOREM

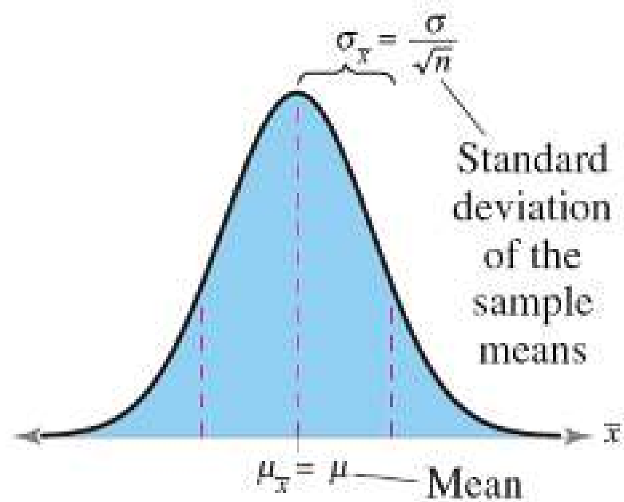
- The Central Limit Theorem forms the *foundation* for the inferential branch of statistics.
- This theorem describes the relationship between the sampling distribution of sample means and the population that the samples are taken from.
- The Central Limit Theorem is an important tool that provides the information you will need to use sample statistics to make inferences about a population mean.

THE CENTRAL LIMIT THEOREM

PART [1]

If samples of size $n \geq 30$ are drawn from *any population* with a mean μ and a standard deviation σ, then the sampling distribution of sample means approximates a **normal distribution**.

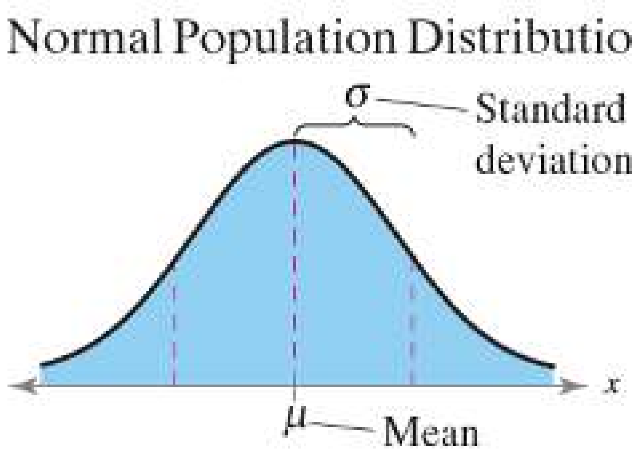


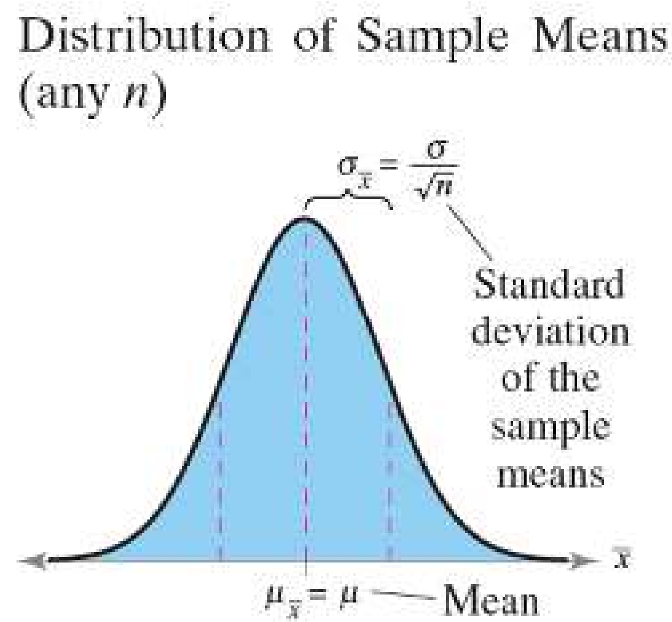


THE CENTRAL LIMIT THEOREM

PART [2]

If the population itself is normally distributed, then the sampling distribution of sample means is normally distributed for any sample size n.





THE CENTRAL LIMIT THEOREM

Example A study analyzed the sleep habits of college students. The study found that the mean sleep time was 6.8 hours, with a standard deviation of 1.4 hours. Random samples of 100 sleep times are drawn from this population, and the mean of each sample is determined. Find the mean and standard deviation of the sampling distribution of sample means. Then sketch a graph of the sampling distribution.

The mean of the sampling distribution is equal to the population mean

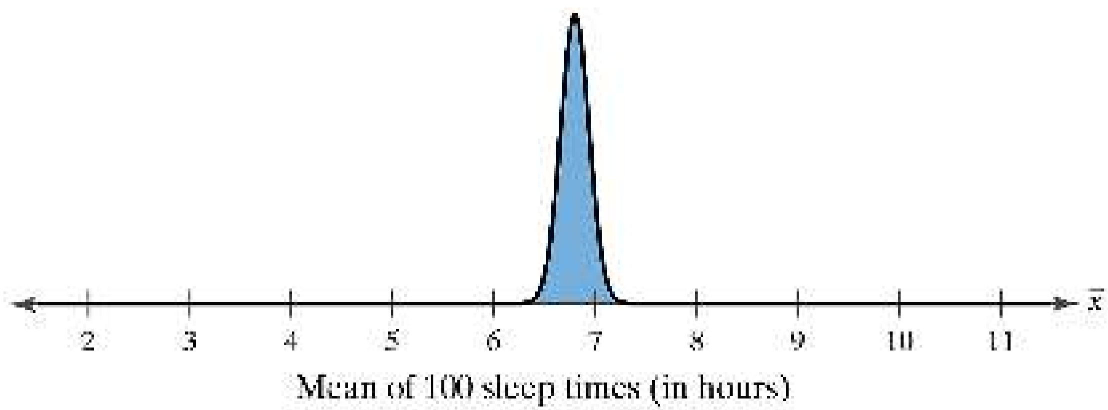
$$\mu_{\bar{x}} = \mu = 6.8$$

The standard error of the mean is equal to the population standard deviation divided by $\sqrt{n}$.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.4}{\sqrt{100}} = 0.14$$

THE CENTRAL LIMIT THEOREM

Example A study analyzed the sleep habits of college students. The study found that the mean sleep time was 6.8 hours, with a standard deviation of 1.4 hours. Random samples of 100 sleep times are drawn from this population, and the mean of each sample is determined. Find the mean and standard deviation of the sampling distribution of sample means. Then sketch a graph of the sampling distribution.



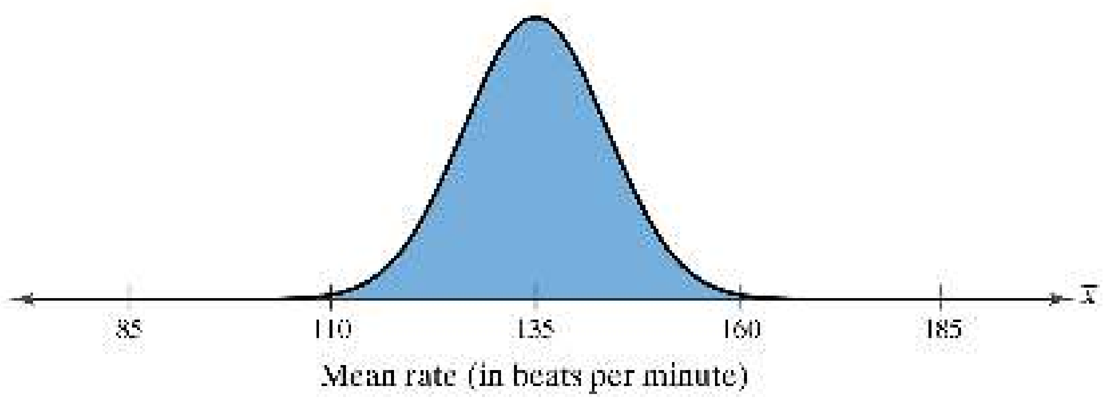
THE CENTRAL LIMIT THEOREM

Example The training heart rates of all 20-years old athletes are normally distributed, with a mean of 135 beats per minute and standard deviation of 18 beats per minute. Random samples of size 4 are drawn from this population, and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling distribution of sample means.

The mean of the sample means	The standard deviation of the sample means
$\mu_{\bar{x}} = \mu = 135 \text{ beats per minute}$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{4}} = 9 \text{ beats per minute}$

THE CENTRAL LIMIT THEOREM

Example The training heart rates of all 20-years old athletes are normally distributed, with a mean of 135 beats per minute and standard deviation of 18 beats per minute. Random samples of size 4 are drawn from this population, and the mean of each sample is determined. Find the mean and standard error of the mean of the sampling distribution. Then sketch a graph of the sampling distribution of sample means.



Course: Biostatistics

Lecture No: [6]

Chapter: [5]

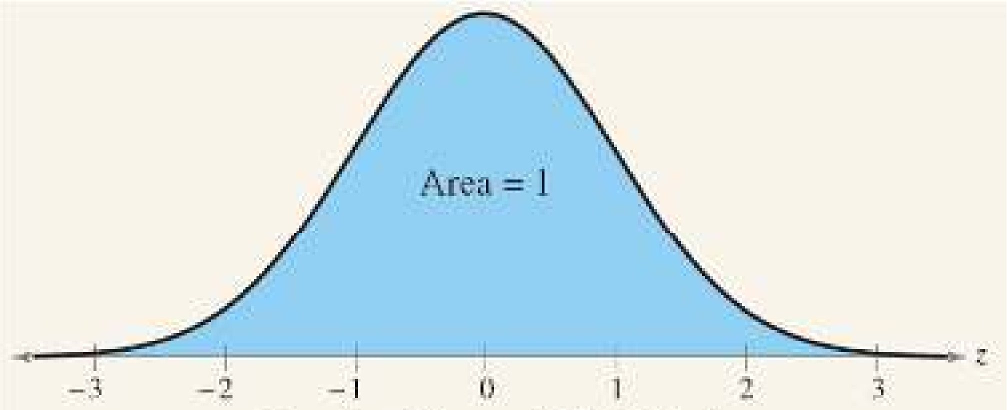
Normal Probability Distributions

Section: [5.*]

Review of the Standard Normal Distribution

THE STANDARD NORMAL DISTRIBUTION (REVIEW)

- The *standard normal distribution* is a normal distribution with a mean of 0 and a standard deviation of 1.
- Denoted by $Z \sim N(0,1)$.



Standard Normal Distribution

THE STANDARD NORMAL DISTRIBUTION (REVIEW)

- There are *infinitely many normal distributions*, each with its own mean and standard deviation.
- You can **transform** an x −value to a z −score using the formula $z = \frac{x-\mu}{\sigma}$.
- After you use the formula, you can use the Standard Normal Table (*Table 4 in Appendix B*)
- The table lists the *cumulative area under the standard normal curve to the left of* z, $P(Z \leq c)$, for z −scores from -3.49 to 3.49

THE STANDARD NORMAL DISTRIBUTION (REVIEW)

Example Using the Standard Normal Table

① $P(Z < 1.15)$

z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279

THE STANDARD NORMAL DISTRIBUTION (REVIEW)

Example Using the Standard Normal Table

(2) $P(Z \leq -0.24)$

z	.09	.08	.07	.06	.05	.04	.03
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880

THE STANDARD NORMAL DISTRIBUTION (REVIEW)

NOTE

① $P(Z \geq c) = 1 - P(Z < c)$

$$= P(Z \leq -c)$$

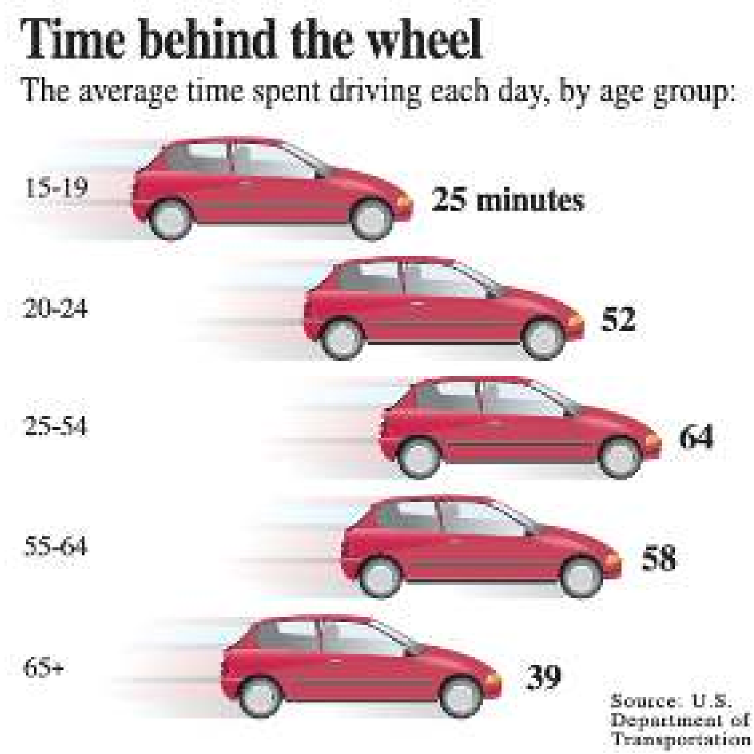
② $P(a \leq Z \leq b) = P(Z \leq b) - P(Z \leq a)$

PROBABILITY AND THE CENTRAL LIMIT THEOREM

NOTE To transform $\bar{x}$ to z −score: $$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Example

The figure at the right shows the lengths of time people spend driving each day. You randomly select 50 drivers ages 15 to 19. What is the probability that the mean time they spend driving each day is between 24.7 and 25.5 minutes? Assume that $\sigma = 1.5$ minutes.



$$\mu_{\bar{x}} = \mu = 25 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{50}} \approx 0.2121$$

PROBABILITY AND THE CENTRAL LIMIT THEOREM

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\mu_{\bar{x}} = 25 \qquad \sigma_{\bar{x}} = 0.2121$$

Example

The figure at the right shows the lengths of time people spend driving each day. You randomly select 50 drivers ages 15 to 19. What is the probability that the mean time they spend driving each day is between 24.7 and 25.5 minutes? Assume that $\sigma = 1.5$ minutes.

$$z_1 = \frac{24.7 - 25}{0.2121} \approx -1.41$$

$$z_2 = \frac{25.5 - 25}{0.2121} \approx 2.36$$

$$P(24.7 \leq \bar{x} \leq 25.5) = P(-1.41 \leq z \leq 2.36)$$

$$= P(z \leq 2.36) - P(z \leq -1.41)$$

$$= 0.9909 - 0.0793 = 0.9116$$

PROBABILITY AND THE CENTRAL LIMIT THEOREM

Example Some college students use credit cards to pay for school-related expenses. For this population, the amount paid is *normally distributed*, with a mean of $1615 and a standard deviation of $550.

1. What is the probability that **a randomly selected** college student, who uses a credit card to pay for school-related expenses, paid less than $1400.

$\mu = 1615$
$\sigma = 550$

$$P(x < 1400) = P\left(z < \frac{1400 - 1615}{550}\right)$$

$$= P(z < -0.39) = 0.3483$$

PROBABILITY AND THE CENTRAL LIMIT THEOREM

Example Some college students use credit cards to pay for school-related expenses. For this population, the amount paid is *normally distributed*, with a mean of \$1615 and a standard deviation of \$550.

2. You **randomly select 25 college students** who use credit cards to pay for school-related expenses. What is the probability that their *mean* amount paid is less than \$1400.

$$\mu = 1615$$
$$\sigma = 550$$

$$\mu_{\bar{x}} = 1615$$
$$\sigma_{\bar{x}} = \frac{550}{\sqrt{25}} = 110$$

$$P(\bar{x} < 1400) = P\left(z < \frac{1400 - 1615}{110}\right)$$

$$= P(z < -1.95) = 0.0256$$

PROBABILITY AND THE CENTRAL LIMIT THEOREM

Example Some college students use credit cards to pay for school-related expenses. For this population, the amount paid is *normally distributed*, with a mean of $1615 and a standard deviation of $550.

Comment Although there is about a 35% chance that a college student who uses a credit card to pay for school-related expenses will pay less than $1400, there is only about a 3% chance that the mean amount a sample of 25 college students will pay is less than $1400. *This is an unusual event.*

Course: Biostatistics Lecture No: [7]

Chapter: [6]

Confidence Intervals

Section: [6.1]

Confidence Intervals for the Mean (σ Known)

IN THIS CHAPTER

- You will *begin* your study of inferential statistics, the second major branch of statistics.
- You will learn how to make a more meaningful estimate by specifying an interval of values on a number line, *together* with a statement of how confident you are that your interval contains the population parameter.

ESTIMATING POPULATION PARAMETERS

- To use *sample statistics* to estimate the value of an *unknown population parameter*.
- A **point estimate** is a single value estimate for a population parameter.
- The most unbiased (غير متحيّز) *point estimate* of the population *mean* μ is the *sample mean* $\bar{x}$.
- A statistic is *unbiased* if it does not overestimate or underestimate the population parameter.

ESTIMATING POPULATION PARAMETERS

- The *validity* (مصداقية) of an estimation method *is increased* when you use a *sample statistic* that is unbiased and has low variability.
- For example, when the standard error $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ of a sample mean is decreased by increasing n, it becomes less variable.

ESTIMATING POPULATION PARAMETERS

Example An economics researcher is collecting data about grocery store employees in a county. The data listed below represents a random sample of the number of hours worked by 30 employees from several grocery stores in the county. Find a point estimate of the population mean μ.

Number of hours					
26	25	32	31	28	28
28	22	28	25	21	40
32	22	25	22	26	24
46	20	35	22	32	48
32	36	38	32	22	19

Since the sample mean of the data is

$$\bar{x} = \frac{\sum x}{n} = \frac{867}{30} = 28.9$$

then the point estimate for the mean number of hours worked by grocery store employees in this county is 28.9 hours.

ESTIMATING POPULATION PARAMETERS

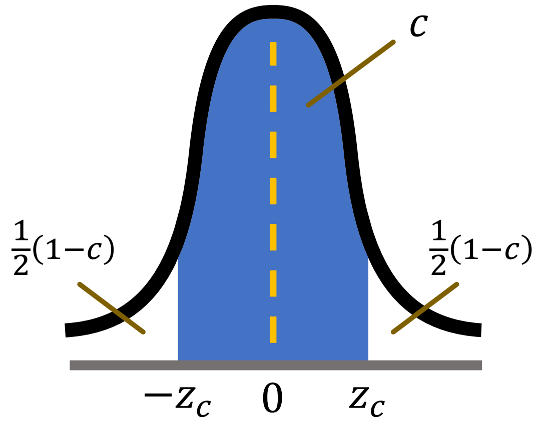
- An *interval estimate* is an interval, or range of values, used to estimate a population parameter.
- To *form* an interval estimate, *use the point estimate as the center of the interval*, and then *add and subtract a margin of error*.
- Before finding a margin of error for an interval estimate, you should first determine how confident you need to be that your interval estimate contains the population mean.

ESTIMATING POPULATION PARAMETERS

- The *level of confidence* c is the probability that the interval estimate contains the population parameter, assuming that the estimation process is repeated many times.
- From the Central Limit Theorem that when $n \geq 30$ or the population is normal, the sampling distribution of sample means is a normal distribution.
- The *level of confidence* c is the area under the standard normal curve between the critical values, $-z_c$ and z_c.

ESTIMATING POPULATION PARAMETERS

- The *level of confidence* c is the area under the standard normal curve between the critical values, $-z_c$ and z_c.



- **Critical values** are values that separate sample statistics that are probable from sample statistics that are improbable, or unusual.

- In this course, you will usually use 90%, 95%, and 99% levels of confidence. Here are the z −scores that correspond to these levels of confidence.

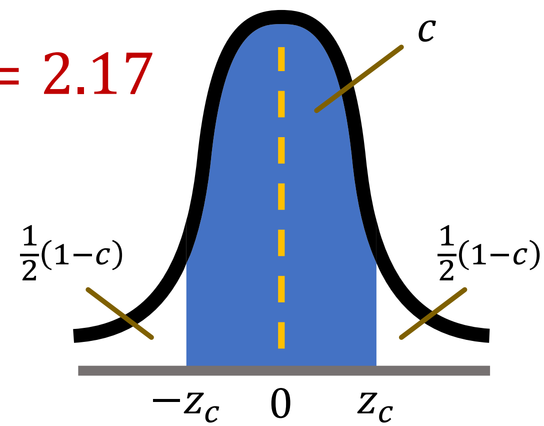
Level of Confidence	z_c
90%	1.645
95%	1.96
99%	2.575

ESTIMATING POPULATION PARAMETERS

Example Find the critical value z_c necessary to construct a confidence interval at the level of confidence:

1. 97% $c = 0.97$ $\dfrac{1-c}{2} = \dfrac{0.03}{2} = 0.0150$ $z_c = 2.17$

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
−2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
−2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
−2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
−2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
−2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
−2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
−2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
−2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179

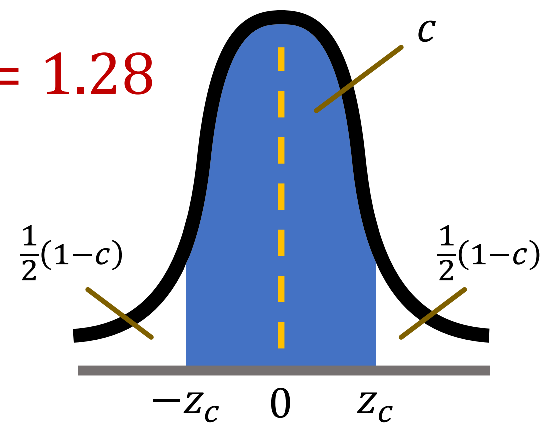


Level of Confidence	z_c
90%	1.645
95%	1.96
99%	2.575

ESTIMATING POPULATION PARAMETERS

Example Find the critical value z_c necessary to construct a confidence interval at the level of confidence:

2. 80% $c = 0.80$ $\frac{1-c}{2} = \frac{0.20}{2} = 0.1000$ $z_c = 1.28$



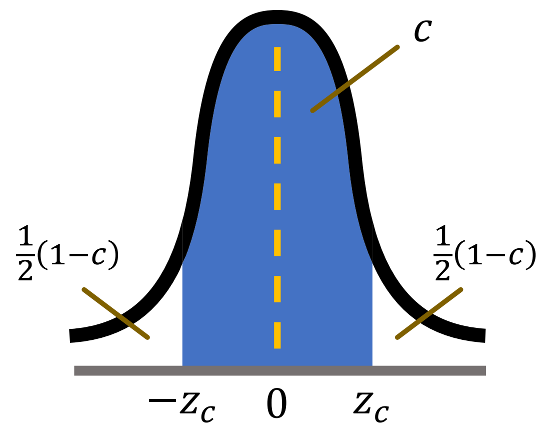
z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−1.7	[illegible]	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
−1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
−1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
−1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
−1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
−1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151

Level of Confidence	z_c
90%	1.645
95%	1.96
99%	2.575

ESTIMATING POPULATION PARAMETERS

Example Find the critical value z_c necessary to construct a confidence interval at the level of confidence:

3 90% $c = 0.90$ $\dfrac{1-c}{2} = \dfrac{0.10}{2} = 0.0500$



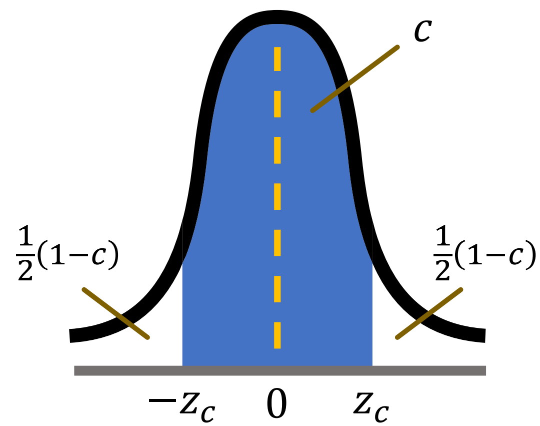
z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
−1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
−1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
−1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808

Level of Confidence	z_c
90%	1.645
95%	1.96
99%	2.575

ESTIMATING POPULATION PARAMETERS

Example Find the critical value z_c necessary to construct a confidence interval at the level of confidence:

3. 90% $c = 0.90$ $$z_c = \frac{1.65 + 1.64}{2} = 1.645$$



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.0013	.0013
−1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
−1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
−1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
−1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808

Level of Confidence	z_c
90%	1.645
95%	1.96
99%	2.575

ESTIMATING POPULATION PARAMETERS

- The difference between the *point estimate* and the *actual parameter* value is called the sampling error.
- The sampling error of the mean is $\bar{x} - \mu$.
- In most cases, μ is unknown, and $\bar{x}$ varies from sample to sample.
- You can calculate a *maximum value for the error* when you know the level of confidence and the sampling distribution.

ESTIMATING POPULATION PARAMETERS

- Given a level of confidence c, the margin of error E is the greatest possible distance between the point estimate and the value of the parameter it is estimating.
- For a population mean μ where σ is known, the margin of error is

$$E = z_c \, \sigma_{\bar{x}} = z_c \, \frac{\sigma}{\sqrt{n}}$$

when these conditions are met:

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or $n \geq 30$.

ESTIMATING POPULATION PARAMETERS

Example An economics researcher is collecting data about grocery store employees in a county. The data listed below represents a random sample of the number of hours worked by 30 employees from several grocery stores in the county. Find a point estimate of the population mean μ.

Number of hours					
26	25	32	31	28	28
28	22	28	25	21	40
32	22	25	22	26	24
46	20	35	22	32	48
32	36	38	32	22	19

Use a 95% confidence level to find the margin of error for the mean number of hours worked by grocery store employees. Assume the population standard deviation is 7.9 hours.

$$\bar{x} = 28.9$$

$$\sigma = 7.9$$

$$c = 0.95$$

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.96 \times \frac{7.9}{\sqrt{30}} \approx 2.83$$

CONFIDENCE INTERVALS FOR A POPULATION MEAN

- Using a *point estimate* and a *margin of error*, you can construct an *interval estimate* of a population parameter.
- A c −confidence interval for a population mean μ is

$$\bar{x} - E < \mu < \bar{x} + E$$

- The probability that the confidence interval contains μ is c, assuming that the estimation process is repeated many times.

CONFIDENCE INTERVALS FOR A POPULATION MEAN

Example A college admissions director wishes to estimate the mean age of all students currently enrolled. In a random sample of 20 students, the mean age is found to be 22.9 years. From past studies, the standard deviation is known to be 1.5 years, and the population is normally distributed. Construct a 90% confidence interval of the population mean age.

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.645 \times \frac{1.5}{\sqrt{20}} \approx 0.6$$

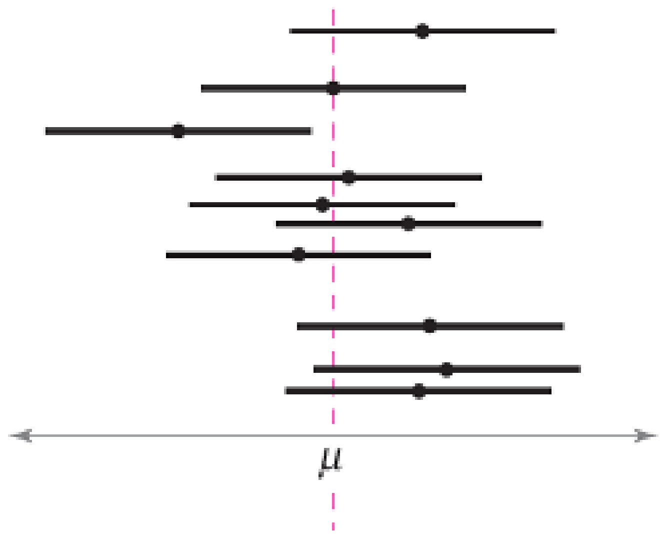
$$\bar{x} = 22.9$$
$$c = 0.90$$
$$\sigma = 1.5$$
$$n = 20$$

$$\left.\begin{aligned} \bar{x} - E &= 22.9 - 0.6 = 22.3 \\ \bar{x} + E &= 22.9 + 0.6 = 23.5 \end{aligned}\right\} \quad 22.3 < \mu < 23.5$$

CONFIDENCE INTERVALS FOR A POPULATION MEAN

Example A college admissions director wishes to estimate the mean age of all students currently enrolled. In a random sample of 20 students, the mean age is found to be 22.9 years. From past studies, the standard deviation is known to be 1.5 years, and the population is normally distributed. Construct a 90% confidence interval of the population mean age.

- "With 90% confidence, the mean is in the interval $(22.3, 23.5)$."
- This means that when many samples is collected and a confidence interval is created for each sample, approximately 90% of these intervals will contain μ.



SAMPLE SIZE

- Given a c −confidence level and a margin of error E, the minimum sample size n needed to estimate the population mean μ is

$$n = \left(\frac{z_c\ \sigma}{E}\right)^2$$

- When σ is unknown, you can estimate it using s, provided you have a preliminary sample with at least 30 members.

SAMPLE SIZE

Example Determine the minimum sample size required when you want to be 99% confident that the sample mean is within two units of the population mean and $\sigma = 1.4$. Assume the population is normally distributed.

$c = 0.99$

$z_c = 2.575$

$E = 2$

$\sigma = 1.4$

$$n = \left(\frac{z_c\,\sigma}{E}\right)^2 = \left(\frac{2.575 \times 1.4}{2}\right)^2 \approx 3.25$$

FINITE POPULATION CORRECTION FACTOR

- In this section, you studied the construction of a confidence interval to estimate a population mean when the population is *large* or *infinite*.
- When a population is *finite*, the formula that determines the standard error of the mean $\sigma_{\bar{x}}$ needs to be *adjusted*.
- If N is the size of the population and n is the size of the sample (where $n \geq 0.05N$), then the standard error of the mean is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

FINITE POPULATION CORRECTION FACTOR

- If N is the size of the population and n is the size of the sample (where $n \geq 0.05N$), then the standard error of the mean is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

- The expression $\sqrt{\frac{N-n}{N-1}}$ is called the **finite population correction factor**.
- The margin of error is

$$E = z_c \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

FINITE POPULATION CORRECTION FACTOR

Example Use the finite population correction factor to construct the confidence interval for the population mean given that

$c = 0.99 \qquad \bar{x} = 8.6 \qquad \sigma = 4.9 \qquad N = 200 \qquad n = 25$

$z_c = 2.575$

$25 \geq (0.05)(200)$

$$E = z_c \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

$$= (2.575) \frac{4.9}{\sqrt{25}} \sqrt{\frac{200-25}{200-1}}$$

$$\approx 2.4$$

$$\bar{x} - E \leq \mu \leq \bar{x} + E$$

$$8.6 - 2.4 \leq \mu \leq 8.6 + 2.4$$

$$6.2 \leq \mu \leq 11.0$$

Course: Biostatistics Lecture No: [8]

Chapter: [6]

Confidence Intervals

Section: [6.2]

Confidence Intervals for the Mean (σ Unknown)

THE t −DISTRIBUTION

- In many real-life situations, the population standard deviation is *unknown*.
- For a random variable that is normally distributed (or approximately normally distributed), you can use a **t −distribution**.
- If the distribution of a random variable x is *approximately normal*, then

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

follows a t −distribution.
- Critical values of t are denoted by t_c.

THE t −DISTRIBUTION

Here are several properties of the t −distribution.

1. The mean, median, and mode of the t −distribution are equal to 0.
2. The t −distribution is bell-shaped and symmetric about the mean.
3. The total area under the t −distribution curve is equal to 1.
4. The tails in the t −distribution are "thicker" than those in the standard normal distribution.
5. The standard deviation of the t −distribution varies with the sample size, but it is greater than 1.

THE t −DISTRIBUTION

Here are several properties of the t −distribution.

6. The t −distribution is a *family of curves*, each determined by a parameter called the *degrees of freedom*.
   - ✓ The degrees of freedom (d.f.) are the number of free choices left after a sample statistic such as $\bar{x}$ is calculated.

$\bar{x} = 2$

x	$x - \bar{x}$
1	-1
2	0
■	1
	0

THE t –DISTRIBUTION

Here are several properties of the t –distribution.

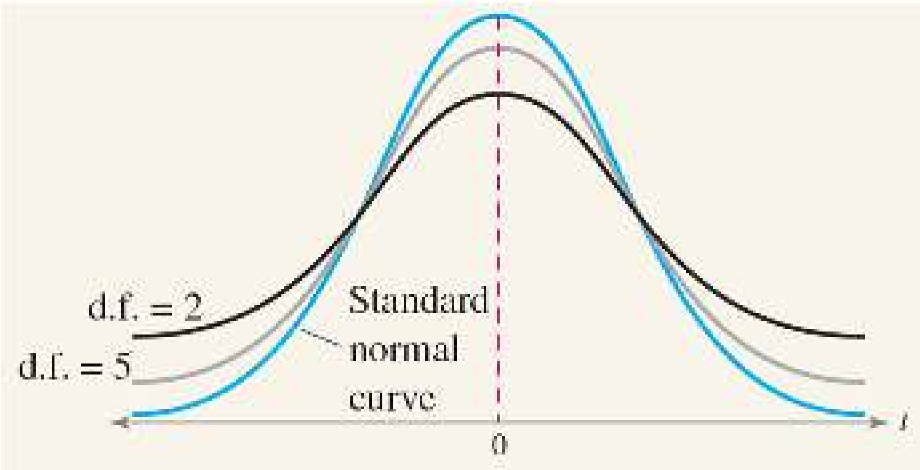
6. The t –distribution is a *family of curves*, each determined by a parameter called the *degrees of freedom*.

- ✓ The degrees of freedom (d.f.) are the number of free choices left after a sample statistic such as x is calculated.
- ✓ When you use a t –distribution to estimate a population mean, the degrees of freedom are equal to one less than the sample size.
- ✓ $d.f. = n - 1$

THE t −DISTRIBUTION

Here are several properties of the t −distribution.

7. As the degrees of freedom increase, the t −distribution approaches the standard normal distribution. After 30 d.f., the t −distribution is close to the standard normal distribution.

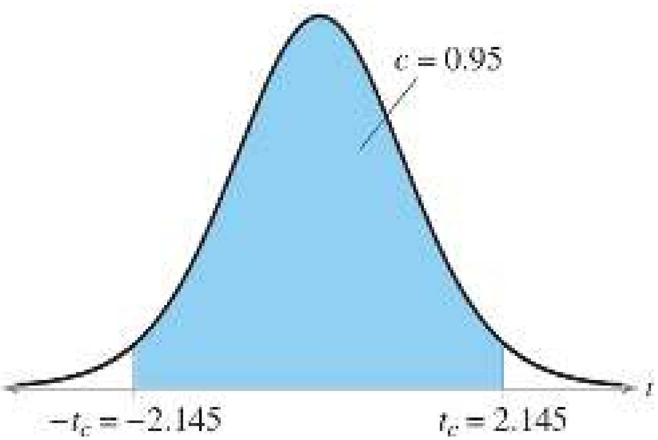


FINDING CRITICAL VALUES OF t

Example Find the critical value t_c for a 95% confidence level when the sample size is 15.

$d.f. = 15 - 1 = 14$

$c = 0.95$ $t_c = 2.145$



d.f.	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921

CONFIDENCE INTERVALS AND t −DISTRIBUTIONS

- Constructing a confidence interval for μ when σ is not known using the t −distribution is like constructing a confidence interval for μ when σ is known using the standard normal distribution.
- Both use a point estimate $\bar{x}$ and a margin of error E.
- When σ is not known, the margin of error E is: $E = t_c \frac{s}{\sqrt{n}}$
- Before using this formula, verify that the sample is random, and either the population is normally distributed or $n \geq 30$.
- s is the sample standard deviation: $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

CONFIDENCE INTERVALS AND t −DISTRIBUTIONS

Example You randomly select 16 coffee shops and measure the temperature of the coffee sold at each. The sample mean temperature is 162.0°F with a sample standard deviation of 10.0°F. Construct a 95% confidence interval for the population mean temperature of coffee sold. Assume the temperatures are approximately normally distributed.

$n = 16$

$\bar{x} = 162$

$s = 10$

$c = 0.95$

CONFIDENCE INTERVALS AND t –DISTRIBUTIONS

Example $n = 16$ $\bar{x} = 162$ $s = 10$ $c = 0.95$ $t_c = 2.131$

$$E = t_c \frac{s}{\sqrt{n}} = 2.131 \cdot \frac{10.0}{\sqrt{16}} \approx 5.3.$$

$$\bar{x} - E \approx 162 - 5.3 = 156.7$$

$$\bar{x} + E \approx 162 + 5.3 = 167.3$$

$$156.7 < \mu < 167.3$$

	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
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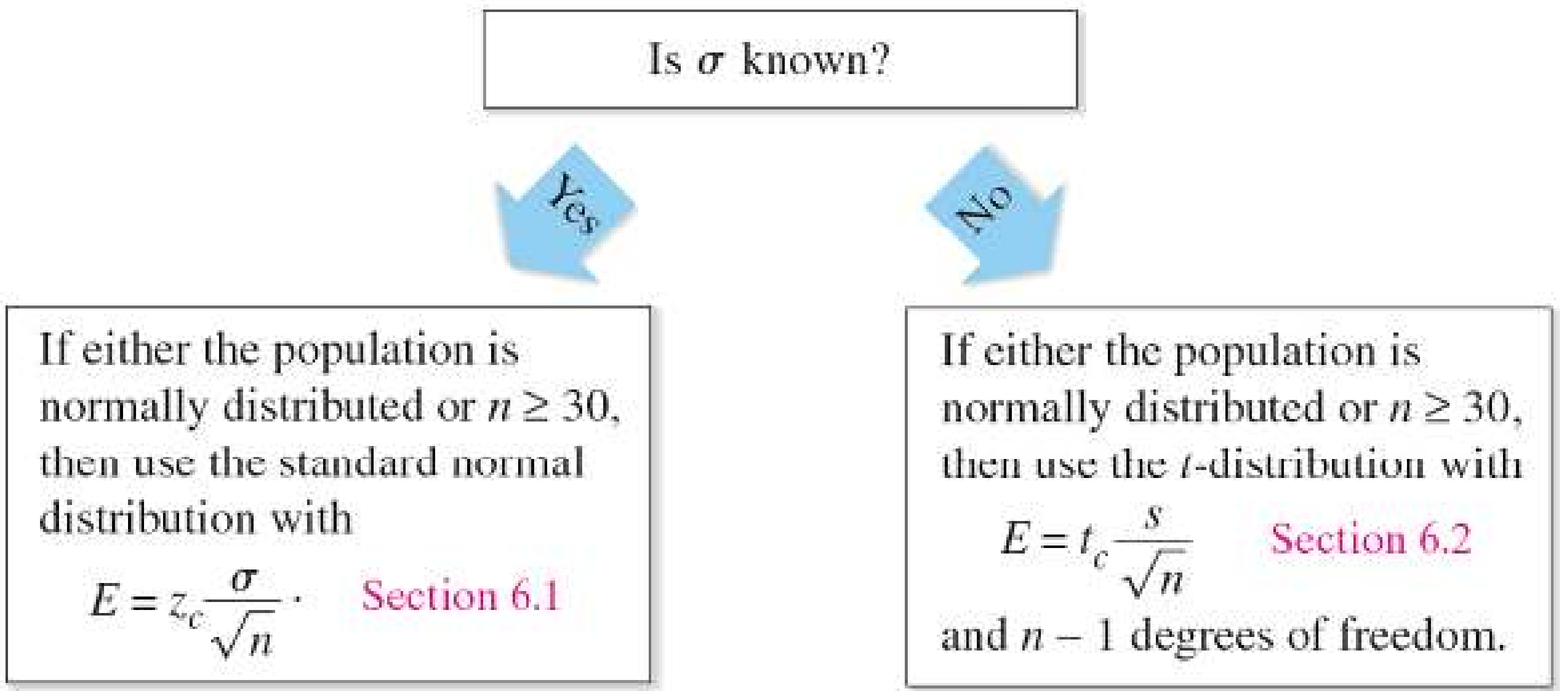
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921

With 95% confidence, you can say that the population mean temperature of coffee sold is between 156.7°F and 167.3°F.

CONFIDENCE INTERVALS AND t −DISTRIBUTIONS

The flowchart describes when to use the standard normal distribution and when to use the t −distribution to construct a confidence interval for a population mean.



Course: Biostatistics

Lecture No: [9]

Chapter: [6]

Confidence Intervals

Section: [6.3]

Confidence Intervals for Population Proportions

POINT ESTIMATE FOR A POPULATION PROPORTION

- Recall that the probability of success in a single trial of a binomial experiment is p.
- This probability is a population proportion.
- The point estimate for p, the population proportion of successes, is given by the proportion of successes in a sample and is denoted by

$$\hat{p} = \frac{x}{n}$$

where x is the *number of successes* in the sample and n is the *sample size*.

POINT ESTIMATE FOR A POPULATION PROPORTION

NOTE The point estimate for the population proportion of failures is $\hat{q} = 1 - \hat{p}$.

Example In a survey of 1000 U.S. teens, 372 said that they own smartphones. Find a point estimate for the population proportion of U.S. teens who own smartphones.

Using $n = 1000$ and $x = 372$:

$$\hat{p} = \frac{372}{1000} = 0.372 = 37.2\%$$

So, the point estimate for the population proportion of U.S. teens who own smartphones is 37.2%.

CONFIDENCE INTERVALS FOR A POPULATION PROPORTION

- A c −confidence interval for a population proportion p is

$$\hat{p} - E < p < \hat{p} + E$$

Where $E = z_c \sqrt{\frac{\hat{p}\hat{q}}{n}}$, assuming that the estimation process is repeated many times.

- A binomial distribution can be approximated by a normal distribution when $np \geq 5$ and $nq \geq 5$.

CONFIDENCE INTERVALS FOR A POPULATION PROPORTION

NOTE When $n\hat{p} \geq 5$, and $n\hat{q} \geq 5$, the sampling distribution of $\hat{p}$ is approximately normal with a mean of $\mu_{\hat{p}} = p$ and a standard error of $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$.

CONSTRUCTING A CONFIDENCE INTERVAL FOR P

Example

The figure is from a survey of 498 U.S. adults. Construct a 99% confidence interval for the population proportion of U.S. adults who think that teenagers are the more dangerous drivers.

$n = 498 \quad \hat{p} = 0.71 \quad \hat{q} = 0.29$

$c = 0.99 \quad z_c = 2.575$

$n\hat{p} = 498 \times 0.71 = 353.58 \geq 5$

$n\hat{q} = 498 \times 0.29 = 144.42 \geq 5$

$$E = z_c \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.575 \sqrt{\frac{(0.71)(0.29)}{498}} \approx 0.052$$

CONSTRUCTING A CONFIDENCE INTERVAL FOR P

Example

The figure is from a survey of 498 U.S. adults. Construct a 99% confidence interval for the population proportion of U.S. adults who think that teenagers are the more dangerous drivers.

$$E = 0.052$$

$$\hat{p} - E = 0.71 - 0.052 = 0.658$$

$$\hat{p} + E = 0.71 + 0.052 = 0.762$$

So, the 99% confidence interval

$$0.658 < p < 0.762$$

FINDING A MINIMUM SAMPLE SIZE

- Given a c −confidence level and a margin of error E, the minimum sample size n needed to estimate the population proportion p is

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2$$

- This formula assumes that you have preliminary estimates of $\hat{p}$ and $\hat{q}$. If not, use $\hat{p} = 0.5$ and $\hat{q} = 0.5$.

FINDING A MINIMUM SAMPLE SIZE

Example You are running a political campaign and wish to estimate, with 95% confidence, the population proportion of registered voters who will vote for your candidate. Your estimate must be accurate within 3% of the population proportion. Find the minimum sample size needed when no preliminary estimate is available.

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2 = (0.5)(0.5)\left(\frac{1.96}{0.03}\right)^2 \approx 1067.11$$

Because n is a decimal, round up to the nearest whole number, 1068.

Course: Biostatistics Lecture No: [10]

Chapter: [6]

Confidence Intervals

Section: [6.4]

Confidence Intervals for Variance and Standard Deviation

THE CHI-SQUARE DISTRIBUTION

- In manufacturing, it is necessary to control the amount that a process varies.
- For instance, an automobile part manufacturer must produce thousands of parts to be used in the manufacturing process. It is important that the parts vary little or not at all.
- How can you measure, and consequently control, the amount of variation in the parts?
- You can start with a point estimate.

THE CHI-SQUARE DISTRIBUTION

- The point estimate for σ^2 is s^2 and the point estimate for σ is s.
- The *most unbiased estimate* for σ^2 is s^2.
- If a random variable x has a normal distribution, then the distribution of

$$\frac{(n-1)s^2}{\sigma^2}$$

forms a **chi-square distribution** for samples of any size $n > 1$.

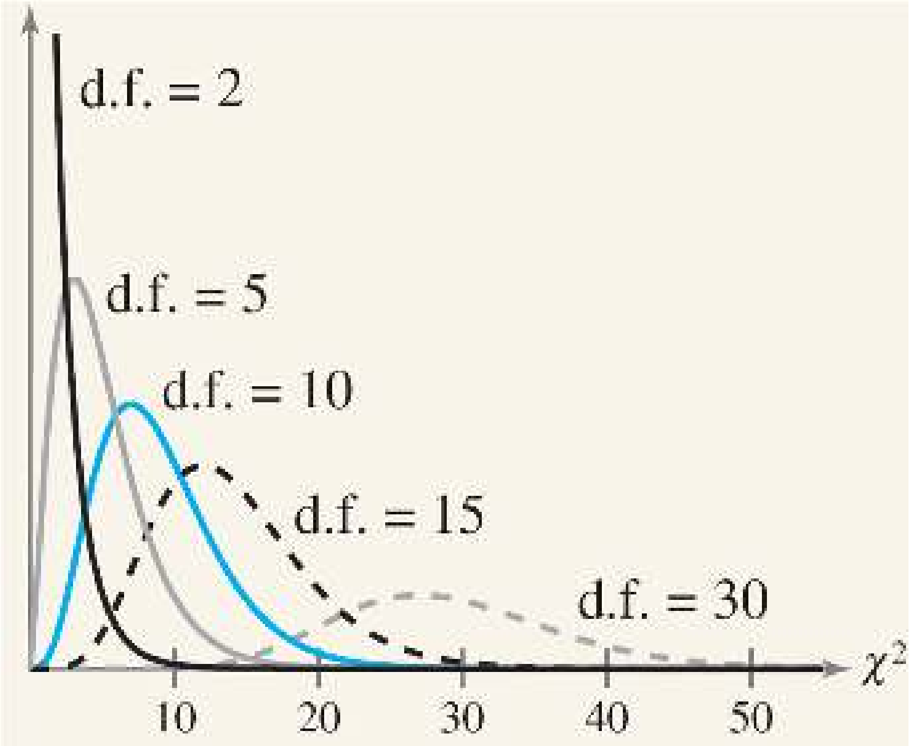
THE CHI-SQUARE DISTRIBUTION

Here are several properties of the chi-square distribution.

1. All values of χ^2 are greater than or equal to 0.
2. The chi-square distribution is a *family of curves*, each determined by the *degrees of freedom*. To form a confidence interval for σ^2, use the chi-square distribution with degrees of freedom equal to one less than the sample size. ($\text{d.f.} = n - 1$)
3. The *total area* under each chi-square distribution curve *is equal to* 1.
4. The chi-square distribution is **positively skewed** and therefore the distribution is **not symmetric**.

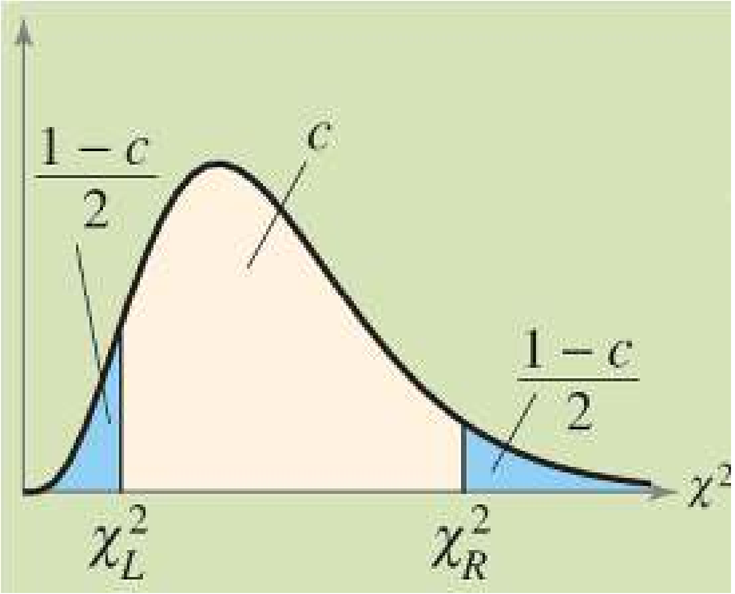
THE CHI-SQUARE DISTRIBUTION

5. The chi-square distribution is different for each number of degrees of freedom. As the degrees of freedom increase, the chi-square distribution approaches a normal distribution.



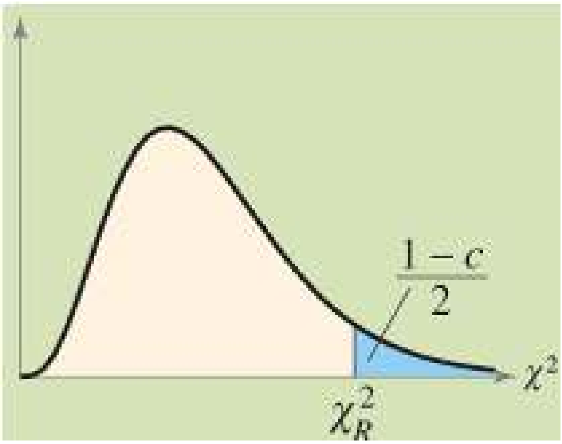
FINDING CRITICAL VALUES FOR CHI-SQUARE

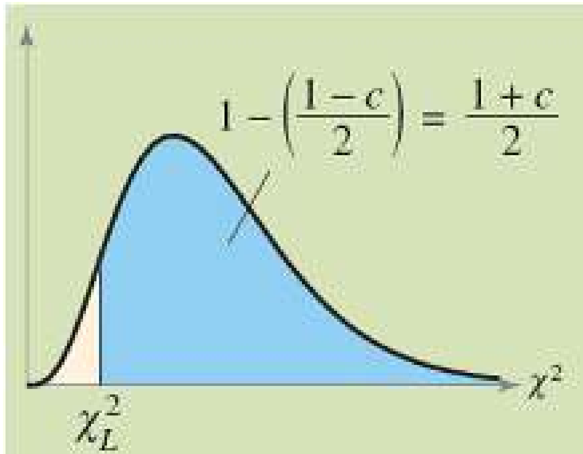
- There are *two critical values* for each level of confidence.
- The value χ_R^2 represents the right-tail critical value and χ_L^2 represents the left-tail critical value.



FINDING CRITICAL VALUES FOR CHI-SQUARE

- Table 6 in Appendix B lists critical values of χ^2 for various degrees of freedom and areas.
- Each area listed in the top row of the table represents the region under the chi-square curve *to the right of the critical value*.





FINDING CRITICAL VALUES FOR CHI-SQUARE

Example Find the critical values χ_R^2 and χ_L^2 for a 95% confidence interval when the sample size is 18.

$$\text{d.f.} = 18 - 1 = 17$$

$$c = 0.95$$

Area to the right of χ_R^2 is $\frac{1-c}{2} = \frac{0.05}{2} = 0.025$

Area to the right of χ_L^2 is $\frac{1+c}{2} = \frac{1.95}{2} = 0.975$

FINDING CRITICAL VALUES FOR CHI-SQUARE

Example Find the critical values χ_R^2 and χ_L^2 for a 95% confidence interval when the sample size is 18.

d.f. $= 17$

Area to the right of χ_R^2 is 0.025

Area to the right of χ_L^2 is 0.975

Degrees of freedom	α							
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170

χ_L^2 χ_R^2

$$\chi_L^2 = 7.564$$

$$\chi_R^2 = 30.191$$

CONFIDENCE INTERVALS FOR σ^2 AND σ

- You can use the critical values χ_L^2 and χ_R^2 to construct confidence intervals for a population variance and standard deviation.
- The best point estimate for the variance is s^2 and the best point estimate for the standard deviation is s.
- Because the chi-square distribution is *not symmetric*, the confidence interval for s^2 cannot be written as $s^2 \pm E$.
- You must do separate calculations for the endpoints of the confidence interval, as shown in the next definition.

CONFIDENCE INTERVALS FOR σ^2 AND σ

The c −confidence intervals for the population variance and standard deviation are shown.

Confidence Interval for σ^2:

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

Confidence Interval for σ:

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

The probability that the confidence intervals contain σ^2 or σ is c, assuming that the estimation process is repeated many times.

NOTE $s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$

CONFIDENCE INTERVALS FOR σ^2 AND σ

Example You randomly select and weigh 30 samples of an allergy medicine. The sample standard deviation is 1.20 milligrams. Assuming the weights are normally distributed, construct 99% confidence intervals for the population variance.

$$\text{d.f.} = 29$$

$$c = 0.99$$

Area to the right of χ^2_R is $\frac{1-c}{2} = \frac{0.01}{2} = 0.005$

Area to the right of χ^2_L is $\frac{1+c}{2} = \frac{1.99}{2} = 0.995$

CONFIDENCE INTERVALS FOR σ^2 AND σ

Example You randomly select and weigh 30 samples of an allergy medicine. The sample standard deviation is 1.20 milligrams. Assuming the weights are normally distributed, construct 99% confidence intervals for the population variance.

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766

CONFIDENCE INTERVALS FOR σ^2 AND σ

Example You randomly select and weigh 30 samples of an allergy medicine. The sample standard deviation is 1.20 milligrams. Assuming the weights are normally distributed, construct 99% confidence intervals for the population variance.

$\chi_L^2 = 13.121$

$\chi_R^2 = 52.336$

Left Endpoint $$\frac{(n-1)s^2}{\chi_R^2} = \frac{(29)(1.2)^2}{52.336} \approx 0.80$$

Right Endpoint $$\frac{(n-1)s^2}{\chi_L^2} = \frac{(29)(1.2)^2}{13.121} \approx 3.18$$

The confidence interval for σ^2 is $0.80 < \sigma^2 < 3.18$

Course: Biostatistics

Lecture No: [11]

Chapter: [7]

Hypothesis Testing with One Sample

Section: [7.1]

Introduction to Hypothesis Testing

WHERE YOU'RE GOING

- ✓ **In Chapter 6:**
  - You learned how to form a confidence interval to estimate a population parameter, such as the proportion of people in JORDAN who agree with a certain statement.
- ✓ **In this chapter:**
  - You will continue your study of inferential statistics.
  - But now, instead of making an estimate about a population parameter, you will learn how to test a claim about a parameter.

HYPOTHESIS TESTS

- ✓ A **hypothesis test** is a process that *uses sample statistics* to test a claim about the value of a population parameter.
- ✓ Researchers in fields such as medicine, psychology, and business rely on hypothesis testing to make informed decisions about new medicines, treatments, and marketing strategies.

HYPOTHESIS TESTS

Example Consider a manufacturer that advertises *its new hybrid car has a mean gas mileage of 50 miles per gallon.*
If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false?

- You cannot test *all* the vehicles.
- But you can still make a reasonable decision about the mean gas mileage by *taking a random sample from the population of vehicles and measuring the mileage of each*.
- If the sample mean differs enough from the advertisement's mean, you can decide that the advertisement is wrong.

HYPOTHESIS TESTS

Example Consider a manufacturer that advertises *its new hybrid car has a mean gas mileage of 50 miles per gallon.*
If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false?

- To test that the mean gas mileage of all hybrid vehicles of this type is $\mu = 50$ miles per gallon, you take a random sample of $n = 30$ vehicles and measure the mileage of each.
- You obtain a sample mean of $\bar{x} = 47$ miles per gallon with a sample standard deviation of $s = 5.5$ miles per gallon.
- Does this indicate that the manufacturer's advertisement is false?

HYPOTHESIS TESTS

Example Consider a manufacturer that advertises *its new hybrid car has a mean gas mileage of 50 miles per gallon.*
If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false?

- To decide, you do something unusual—*you assume the advertisement is correct*! That is, you assume that $\mu = 50$.
- Then, you examine the sampling distribution of sample means (with $n = 30$) taken from a population in which $\mu = 50$ and $\sigma = 5.5$.
- From the Central Limit Theorem, you know this sampling distribution is normal with a mean of 50 and standard error of $\sigma_{\bar{x}} = \frac{5.5}{\sqrt{30}} \approx 1$

HYPOTHESIS TESTS

Example Consider a manufacturer that advertises *its new hybrid car has a mean gas mileage of 50 miles per gallon.*
If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false?

- So $P(\bar{x} \leq 47) = P\left(z \leq \frac{47-50}{1}\right) = P(z \leq -3) = 0.0013.$
- This is an unusual event! Because

$$P(x \leq 47) = P\left(z \leq \frac{47-50}{5.5}\right) = P(z \leq -0.55) = 0.2912$$

- Your assumption that the company's advertisement is correct has led you to an improbable result.

HYPOTHESIS TESTS

Example Consider a manufacturer that advertises *its new hybrid car has a mean gas mileage of 50 miles per gallon.*

If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false?

- So, either you had a very unusual sample, or the advertisement is probably false.
- The logical conclusion is that the advertisement is probably false.

STATING A HYPOTHESIS

- A statement about a population parameter is called a *statistical hypothesis*.
- To test a population parameter, you should *carefully state a pair of hypotheses*; one that represents the claim and the other, its complement.
- When one of these hypotheses is false, the other must be true.
- Either hypothesis, the **null hypothesis** or the **alternative hypothesis**, may represent the original claim.

STATING A HYPOTHESIS

- A **null hypothesis** $\boldsymbol{H_0}$ is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
- The **alternative hypothesis** $\boldsymbol{H_a}$ is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$.
- The symbol H_0 is read as "H sub-zero" or "H naught" and H_a is read as "H sub-a."

STATING A HYPOTHESIS

- To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement.
- Then, write its complement.

$$\begin{aligned} H_0 &: \mu \leq k \\ H_a &: \mu > k \end{aligned} \qquad \begin{aligned} H_0 &: \mu = k \\ H_a &: \mu \neq k \end{aligned} \qquad \begin{aligned} H_0 &: \mu \geq k \\ H_a &: \mu < k \end{aligned}$$

STATING A HYPOTHESIS

Verbal Statement H_0 *The mean is . . .*	Mathematical Statements	Verbal Statement H_a *The mean is . . .*
. . . greater than or equal to k. *. . . at least k.* *. . . not less than k.*	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$	*. . . less than k.* *. . . below k.* *. . . fewer than k.*
. . . less than or equal to k. *. . . at most k.* *. . . not more than k.*	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$	*. . . greater than k.* *. . . above k.* *. . . more than k.*
. . . equal to k. *. . . k.* *. . . exactly k.*	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$	*. . . not equal to k.* *. . . different from k.* *. . . not k.*

STATING THE NULL AND ALTERNATIVE HYPOTHESES

Example Write the claim as a mathematical statement. State the null and alternative hypotheses and identify which represents the claim.

1. A school publicizes (يعلن) that the proportion of its students who are involved in at least one extracurricular activity (نشاط غير منهجي) is 61%.

$$H_0 \quad : \quad p = 0.61$$ **(CLAIM)**

$$H_a \quad : \quad p \neq 0.61$$

STATING THE NULL AND ALTERNATIVE HYPOTHESES

Example Write the claim as a mathematical statement. State the null and alternative hypotheses and identify which represents the claim.

2. A car dealership announces that the mean time for an oil change is less than 15 minutes.

$$H_0 \quad : \quad \mu \geq 15$$
$$H_a \quad : \quad \mu < 15 \quad \textbf{(CLAIM)}$$

STATING THE NULL AND ALTERNATIVE HYPOTHESES

Example Write the claim as a mathematical statement. State the null and alternative hypotheses and identify which represents the claim.

3. A company advertises that the mean life of its furnaces (الأفران) is more than 18 years.

$$H_0 \quad : \quad \mu \leq 18$$
$$H_a \quad : \quad \mu > 18$$ **(CLAIM)**

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

- No matter which hypothesis represents the claim, *you always begin a hypothesis test by assuming that the equality condition in the null hypothesis is true*.
- So, when you perform a hypothesis test, you make one of two decisions:

1. reject the null hypothesis, or
2. fail to reject the null hypothesis.

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

- Because your decision is based on a sample rather than the entire population, there is always the possibility you will make the wrong decision.
- You might *reject a null hypothesis when it is actually true*.
- Or, you might *fail to reject a null hypothesis when it is actually false*.

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

- A **type I error occurs** if the null hypothesis is rejected when it is true.
- A **type II error occurs** if the null hypothesis is not rejected when it is false.

	Truth of H_0	
Decision	**H_0 is true.**	**H_0 is false.**
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

Example The *U.S. Department of Agriculture* limit for salmonella contamination (تلوث) for chicken is 20%. A meat inspector (مفتش اللحوم) reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which error is more serious?

- A type I error will occur when the *actual* proportion of contaminated chicken is less than or equal to 0.2, *but you reject* H_0.
- A type II error will occur when the *actual* proportion of contaminated chicken is greater than 0.2, but you *do not reject* H_0.

TYPES OF ERRORS AND LEVEL OF SIGNIFICANCE

Example The *U.S. Department of Agriculture* limit for salmonella contamination (تلوث) for chicken is 20%. A meat inspector (مفتش اللحوم) reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which error is more serious?

- With a type I error, you might create a health scare and hurt the sales of chicken producers who were actually meeting the USDA limits.
- With a type II error, you could be allowing chicken that exceeded the USDA contamination limit to be sold to consumers.
- A type II error is more serious because it could result in sickness or even death.

LEVEL OF SIGNIFICANCE

- You will *reject* the null hypothesis when the sample statistic from the sampling distribution is *unusual*.
- When statistical tests are used, an unusual event is sometimes required to have a probability of 0.10 or less, 0.05 or less, or 0.01 or less.
- Because there is variation from sample to sample, there is always a possibility that you will reject a null hypothesis when it is actually true.
- You can decrease the probability of this happening by lowering the **level of significance**.

LEVEL OF SIGNIFICANCE

- In a hypothesis test, the *level of significance* is *your maximum allowable probability of making a type I error*. It is denoted by α.
- By setting the level of significance at a small value, you are saying that *you want the probability of rejecting a true null hypothesis to be small*.
- Three commonly used levels of significance are $\alpha = 0.10$, $\alpha = 0.05$, and $\alpha = 0.01$.

LEVEL OF SIGNIFICANCE

- The probability of a type II error is denoted by β.
- When you *decrease* α (the maximum allowable probability of making a type I error), you are *likely to be increasing* β.
- The value $1 - \beta$ is called the **power of the test**.
- It represents the *probability of rejecting the null hypothesis when it is false.*
- The value of the power is **difficult** (and sometimes impossible) to find in most cases.

STATISTICAL TESTS AND P −VALUES

- After stating the null and alternative hypotheses and specifying the level of significance, the next step in a hypothesis test is to obtain a random sample from the population and calculate the sample statistic (such as $\bar{x}$, $\hat{p}$, or s^2) corresponding to the parameter in the null hypothesis (such as μ, p, or σ^2).
- This sample statistic is called the *test statistic*.

STATISTICAL TESTS AND P –VALUES

- With the assumption that the null hypothesis is true, the test statistic is then converted to a *standardized test statistic*, such as z, t, or χ^2.
- The standardized test statistic is used in making the decision about the null hypothesis.

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 7.2, σ known), t (Section 7.3, σ unknown)
p	$\hat{p}$	z (Section 7.4)
σ^2	s^2	χ^2 (Section 7.5)

STATISTICAL TESTS AND P −VALUES

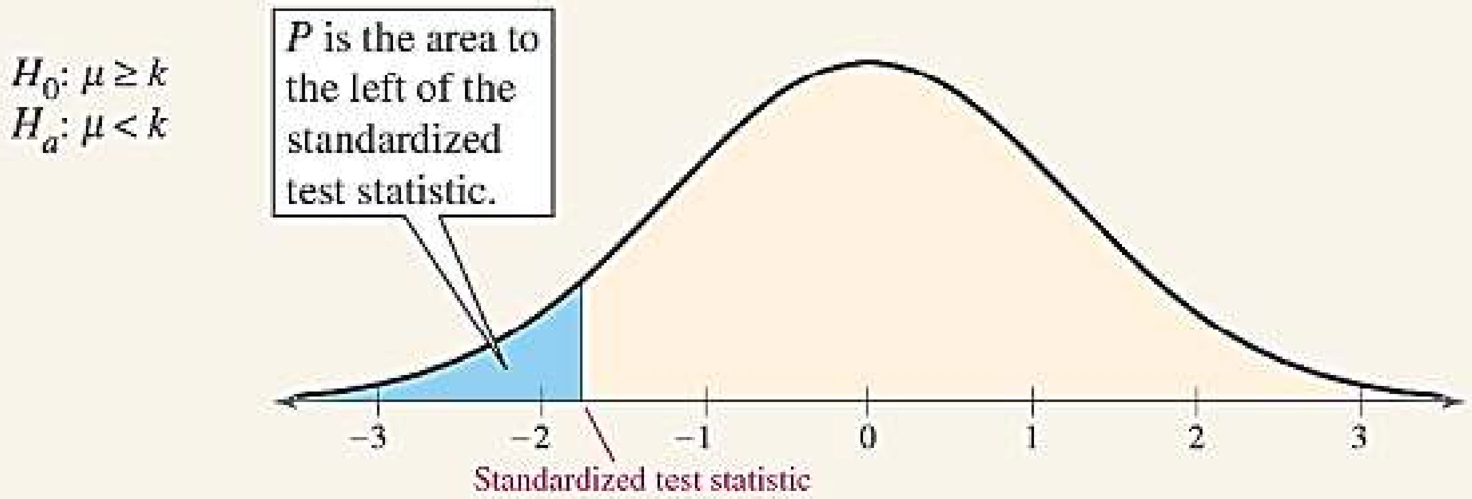
- ✓ If the null hypothesis is true, then a P −value (or probability value) of a hypothesis test is *the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data*.
- ✓ The P −value of a hypothesis test depends on the nature of the test.

STATISTICAL TESTS AND P −VALUES

- ✓ There are *three types of hypothesis tests*—**left-tailed**, **right-tailed**, and **two-tailed**.
- ✓ The type of test depends on the location of the region of the sampling distribution that favors a rejection of H_0.
- ✓ This region is indicated by the alternative hypothesis.

STATISTICAL TESTS AND P −VALUES

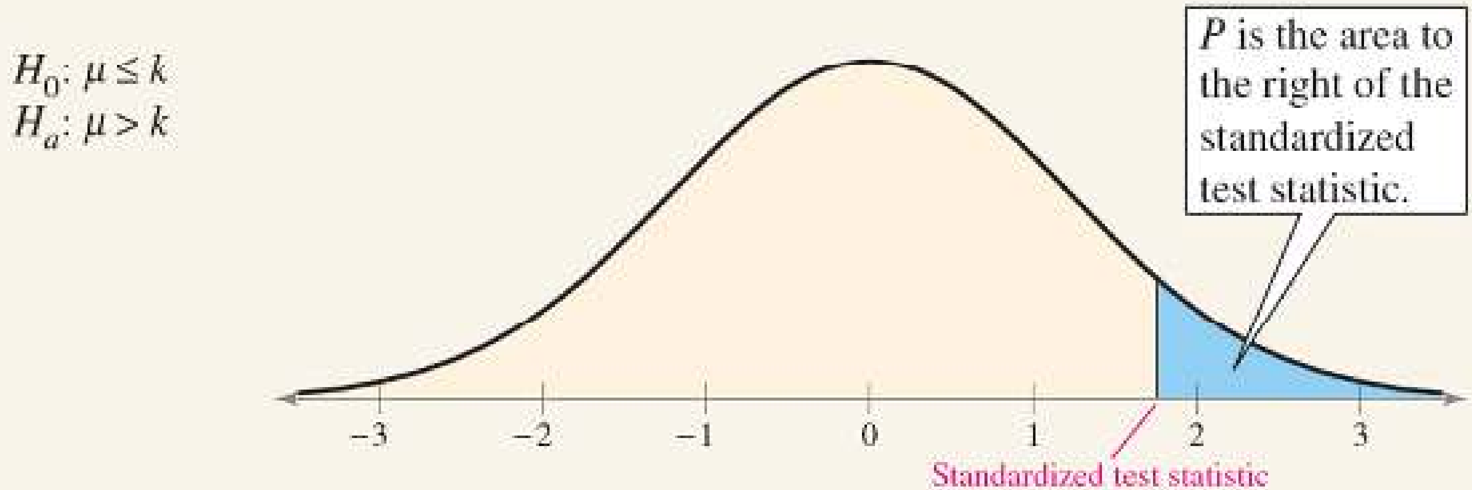
1. If the alternative hypothesis H_a contains the less-than inequality symbol $(<)$, then the hypothesis test is a **left-tailed test.**



Left-Tailed Test

STATISTICAL TESTS AND P −VALUES

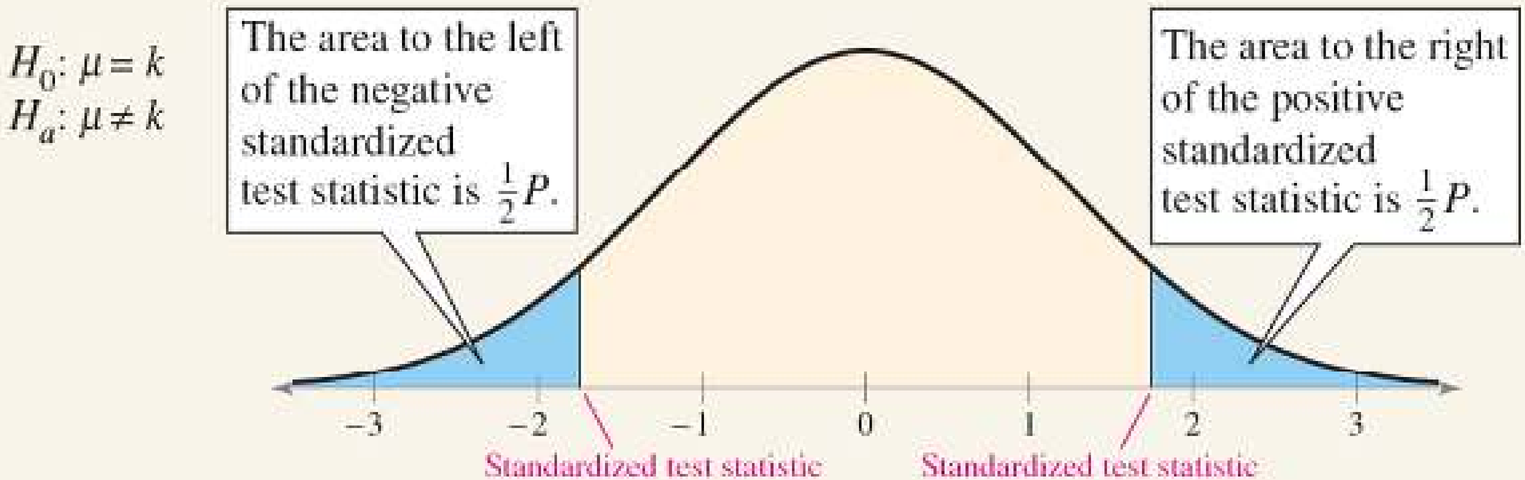
2. If the alternative hypothesis H_a contains the greater-than inequality symbol ($>$), then the hypothesis test is a **right-tailed test.**



Right-Tailed Test

STATISTICAL TESTS AND P −VALUES

3. If the alternative hypothesis H_a contains the not-equal-to symbol ($\neq$), then the hypothesis test is a **two-tailed test.** In a two-tailed test, each tail has an area of $\frac{1}{2}P$.



Two-Tailed Test

STATISTICAL TESTS AND P −VALUES

- The smaller the P −value of the test, the more evidence there is to reject the null hypothesis.
- A very small P −value indicates an unusual event.
- Remember that even a very low P −value does not constitute proof (تشكل دليلاً) that the null hypothesis is false, only that it is probably false.

MAKING A DECISION AND INTERPRETING THE DECISION

DECISION RULE BASED ON P-VALUE

To use a P −value to make a decision in a hypothesis test, compare the P −value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

MAKING A DECISION AND INTERPRETING THE DECISION

The table will help you interpret your decision.

	Claim	
Decision	**Claim is H_0.**	**Claim is H_a.**
Reject H_0.	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

STEPS FOR HYPOTHESIS TESTING

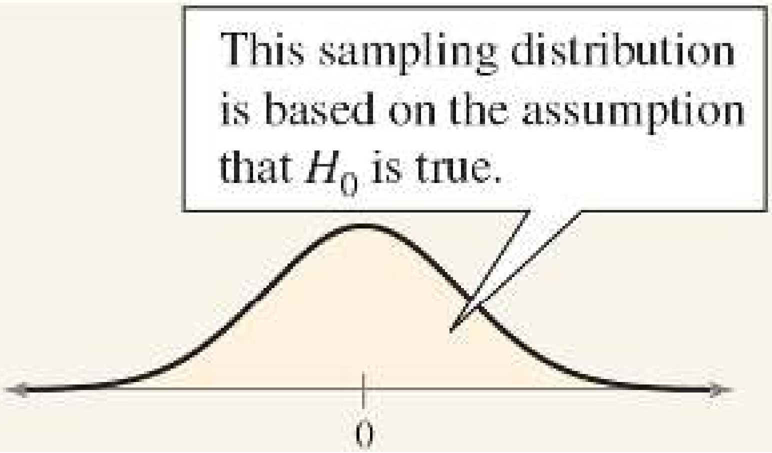
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

H_0: ? H_a: ?

2. Specify the level of significance.

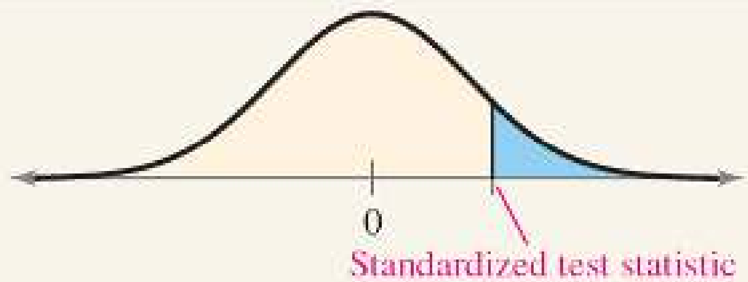
$\alpha =$?

3. Determine the standardized sampling distribution and sketch its graph.



STEPS FOR HYPOTHESIS TESTING

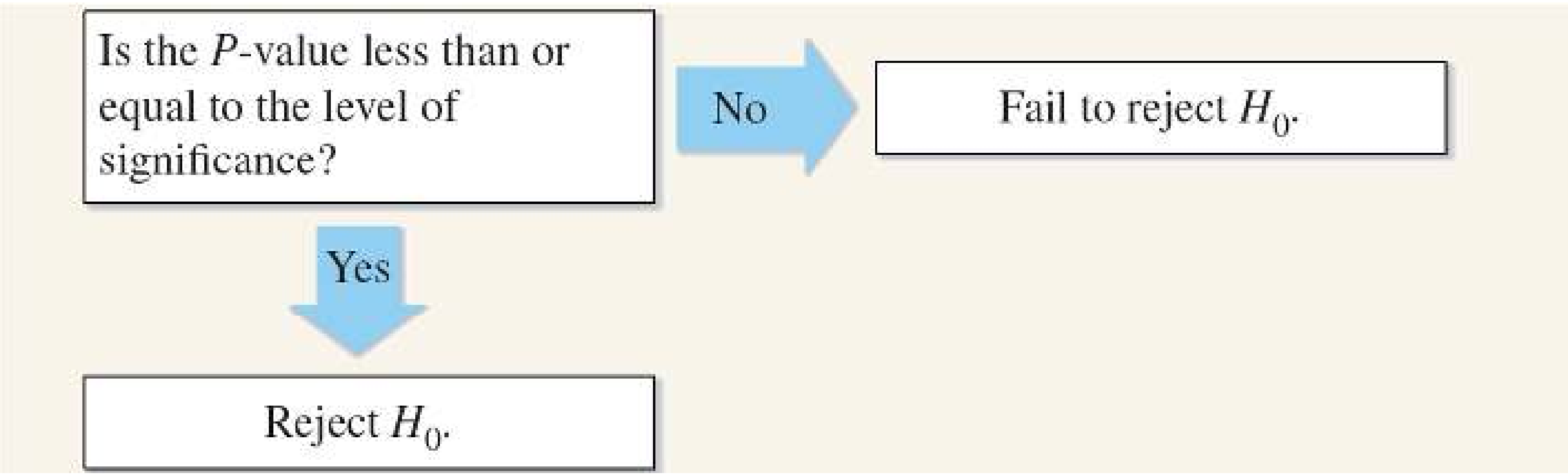
4. Calculate the test statistic and its corresponding standardized test statistic. Add it to your sketch.



5. Find the *P*-value.

6. Use this decision rule.

STEPS FOR HYPOTHESIS TESTING



7. Write a statement to interpret the decision in the context of the original claim.

Course: Biostatistics

Lecture No: [13]

Chapter: [7]

Hypothesis Testing with One Sample

Section: [7.2]

Hypothesis Testing for the Mean (σ Known)

USING P–VALUES TO MAKE DECISIONS

DECISION RULE BASED ON P-VALUE

To use a P −value to make a decision in a hypothesis test, compare the P −value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

USING P –VALUES TO MAKE DECISIONS

Example The P –value for a hypothesis test is $P = 0.0237$. What is your decision when the level of significance is

1. $\alpha = 0.05$

   Because $0.0237 < 0.05$, you reject the null hypothesis.

2. $\alpha = 0.01$

   Because $0.0237 > 0.01$, you fail to reject the null hypothesis.

USING P −VALUES TO MAKE DECISIONS

FINDING THE P-VALUE FOR A HYPOTHESIS TEST

After determining the hypothesis test's standardized test statistic and the standardized test statistic's corresponding area, do one of the following to find the P −value.

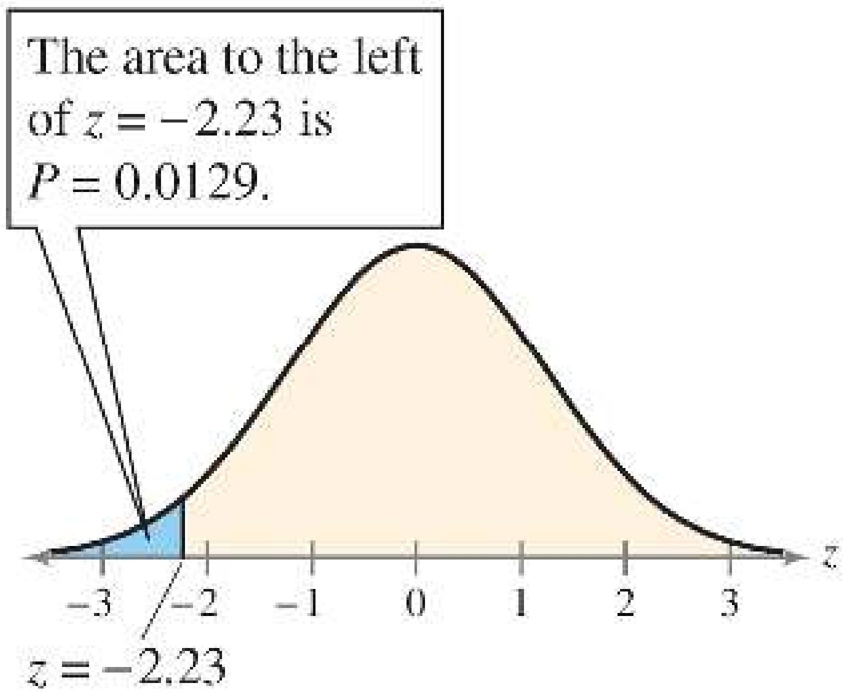
- For a left-tailed test, $P =$ (*Area in left tail*).
- For a right-tailed test, $P =$ (*Area in right tail*).
- For a two-tailed test, $P =$ 2(*Area in tail of standardized test statistic*).

USING P –VALUES TO MAKE DECISIONS

Example Find the $P-$value for a left-tailed hypothesis test with a standardized test statistic of $z = -2.23$. Decide whether to reject H_0 when the level of significance is $\alpha = 0.01$.

$$P = P(z < -2.23) = 0.0129$$

Because the P –value of 0.0129 is greater than 0.01, you fail to reject H_0.

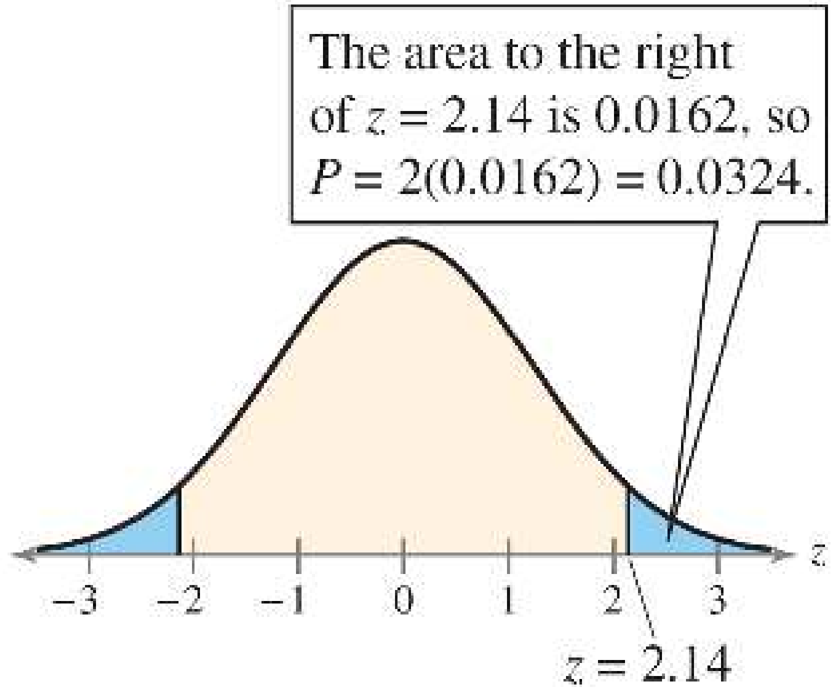


USING P −VALUES TO MAKE DECISIONS

Example Find the P−value for a two-tailed hypothesis test with a standardized test statistic of $z = 2.14$. Decide whether to reject H_0 when the level of significance is $\alpha = 0.05$.

$$P = 2P(z < -2.14)$$
$$= 2(0.0162) = 0.0324$$

Because the P −value of 0.0324 is less than 0.05, you reject H_0.



USING P −VALUES FOR A z −TEST

- The z −test for a mean μ is a statistical test for a population mean.
- The **test statistic** is the sample mean $\bar{x}$.
- The **standardized test statistic** is $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ when these conditions are met.
  - ✓ The sample is random.
  - ✓ At least one of the following is true: The population is normally distributed or $n \geq 30$.
- Recall that $\sigma/\sqrt{n}$ is the standard error of the mean, $\sigma_{\bar{x}}$.

USING P –VALUES FOR A z –TEST

Using P-Values for a z-Test for a Mean μ (σ Known)

IN WORDS	IN SYMBOLS
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.

USING P −VALUES FOR A z −TEST

4. Find the standardized test statistic.	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
5. Find the area that corresponds to z.	Use Table 4 in Appendix B.

6. Find the P-value.

a. For a left-tailed test, $P = (\text{Area in left tail})$.

b. For a right-tailed test, $P = (\text{Area in right tail})$.

c. For a two-tailed test, $P - 2(\text{Area in tail of standardized test statistic})$.

7. Make a decision to reject or fail to reject the null hypothesis.	If $P \leq \alpha$, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

HYPOTHESIS TESTING USING A P −VALUE

Example In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a P −value.

Claim Mean pit stop time is less than 13 seconds

$\mu = 13$ $\quad \bar{x} = 12.9$ $\quad \alpha = 0.01$

$n = 32$ $\quad \sigma = 0.19$

HYPOTHESIS TESTING USING A P –VALUE

Example **Claim** Mean pit stop time is less than 13 seconds

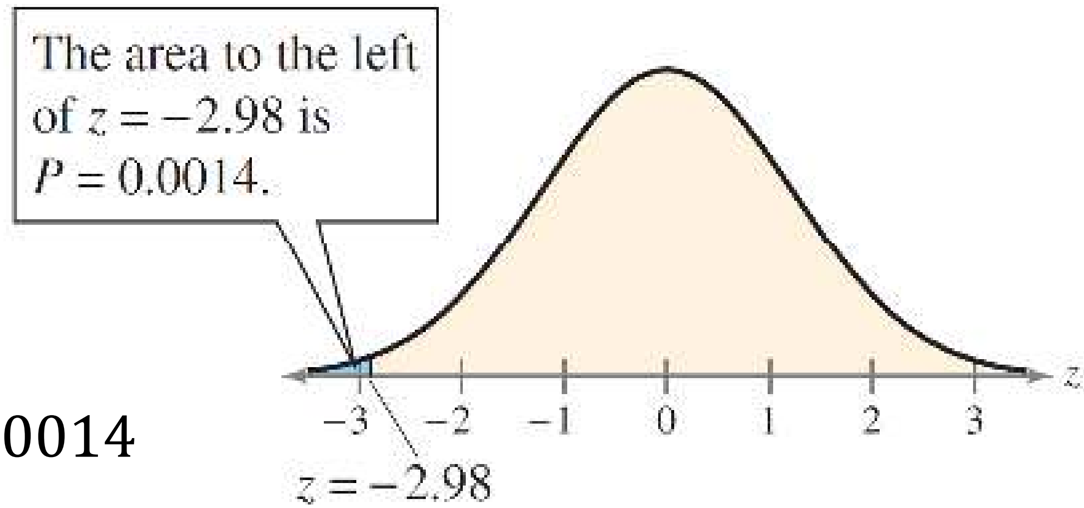
$H_0 \quad : \quad \mu \geq 13$

$H_a \quad : \quad \mu < 13$

(Left Tailed)

$\mu = 13 \qquad \bar{x} = 12.9 \qquad \alpha = 0.01$

$n = 32 \qquad \sigma = 0.19$



The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{12.9 - 13}{0.19/\sqrt{32}} \approx -2.98$$

$$\therefore P - \text{value} = P(z < -2.98) = 0.0014$$

HYPOTHESIS TESTING USING A P −VALUE

Example **Claim** Mean pit stop time is less than 13 seconds

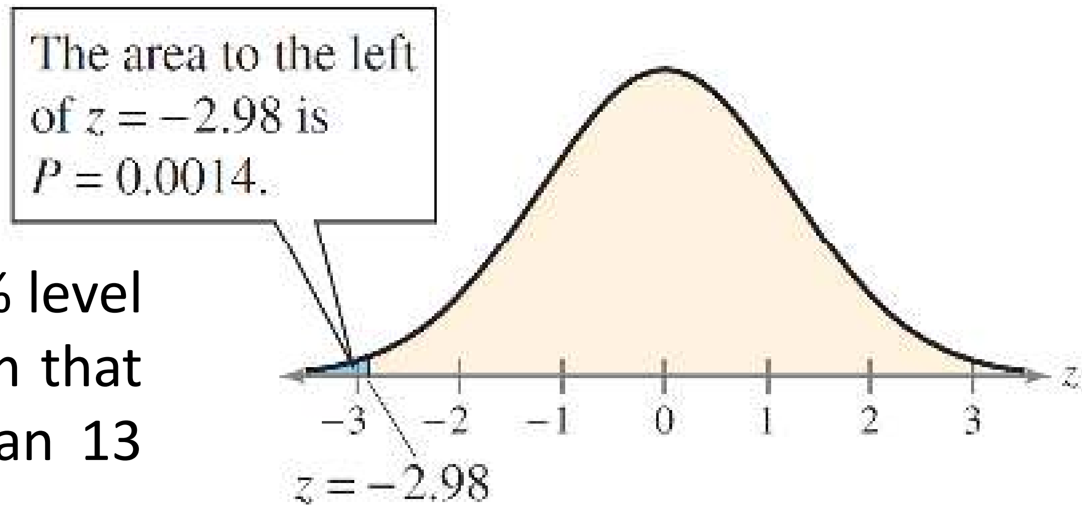
$H_0 \; : \; \mu \geq 13$
$H_a \; : \; \mu < 13$

(Left Tailed)

$$P - \text{value} = 0.0014 \qquad \alpha = 0.01$$

Since $0.0014 < 0.01$ we reject H_0

There is enough evidence at the 1% level of significance to support the claim that the mean pit stop time is less than 13 seconds.



HYPOTHESIS TESTING USING A P –VALUE

Example According to a study, the mean cost of bariatric (weight loss) surgery is \$21,500. You think this information is incorrect. You randomly select 25 bariatric surgery patients and find that the mean cost for their surgeries is \$20,695. From past studies, the population standard deviation is known to be \$2250 and the population is normally distributed. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a P –value.

Claim Mean cost of bariatric (weight loss) surgery is not \$21,500

$\mu = 21500$ $\bar{x} = 20695$ $\alpha = 0.05$

$n = 25$ $\sigma = 2250$

HYPOTHESIS TESTING USING A P –VALUE

Example **Claim** Mean cost of bariatric (weight loss) surgery is not $21,500

$H_0 : \mu = 21500$
$H_a : \mu \neq 21500$

(Two Tailed)

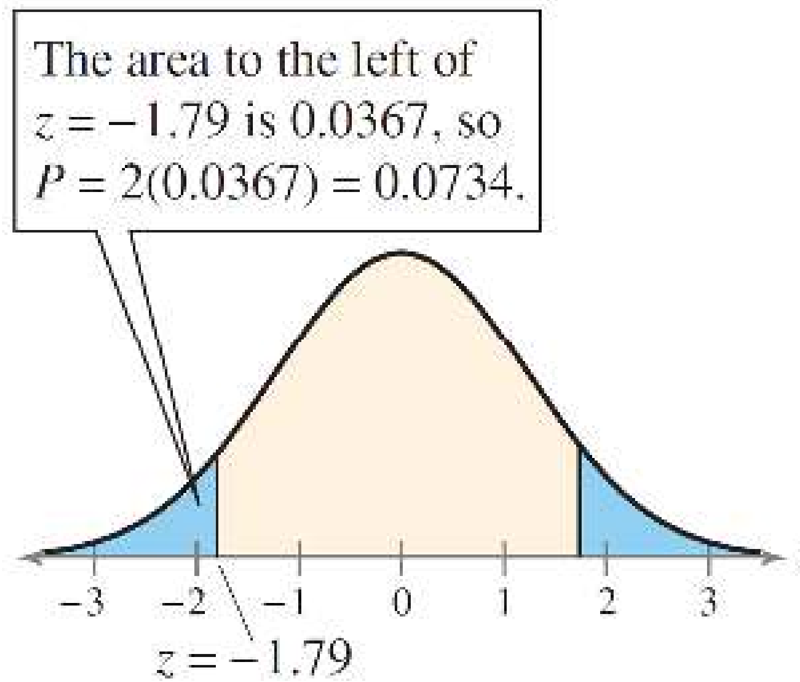
$\mu = 21500 \quad \bar{x} = 20695 \quad \alpha = 0.05$

$n = 25 \quad \sigma = 2250$

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{20695 - 21500}{2250/\sqrt{25}} \approx -1.79$$

$$\therefore P - \text{value} = 2P(z < -1.79) = 2(0.0367) = 0.0734$$



HYPOTHESIS TESTING USING A P −VALUE

Example **Claim** Mean cost of bariatric (weight loss) surgery is not $21,500

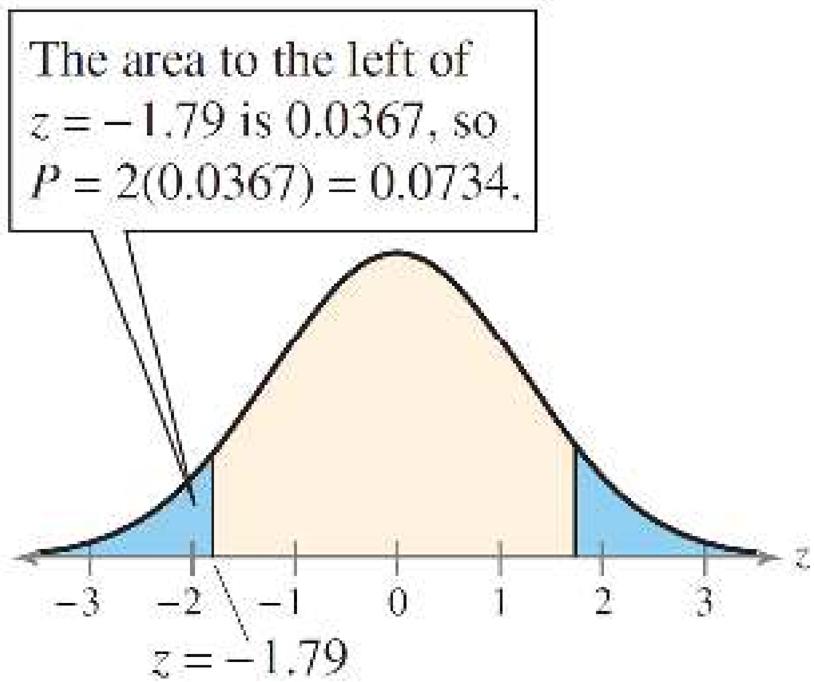
$H_0 \;:\; \mu = 21500$
$H_a \;:\; \mu \neq 21500$

(Two Tailed)

$P - \text{value} = 0.0734 \qquad \alpha = 0.05$

Since $0.0734 > 0.05$ we fail to reject H_0

There is not enough evidence at the 5% level of significance to support the claim that the mean cost of bariatric surgery is different from $21,500.



REJECTION REGIONS AND CRITICAL VALUES

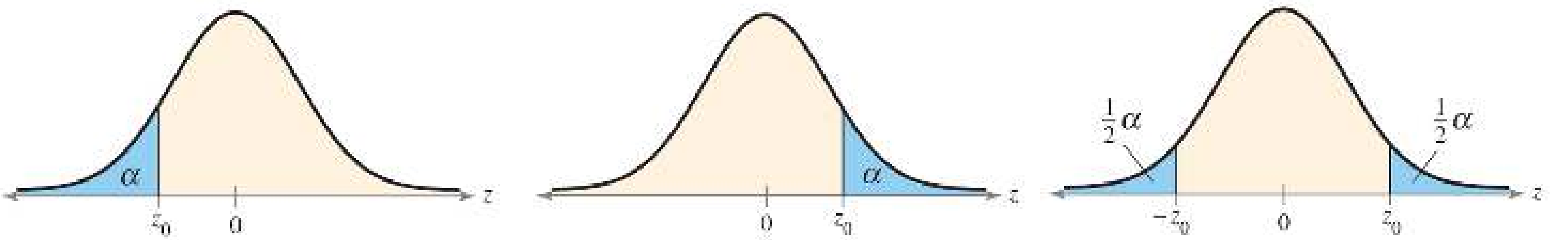
- Another method to decide whether to reject the null hypothesis is to determine whether the standardized test statistic falls within a range of values called the *rejection region* of the sampling distribution.
- A **rejection region** (or *critical region*) of the sampling distribution is the range of values for which the null hypothesis is not probable.
- If a standardized test statistic falls in this region, then the null hypothesis is rejected.
- A critical value z_0 separates the rejection region from the nonrejection region.

REJECTION REGIONS AND CRITICAL VALUES

Finding Critical Values in the Standard Normal Distribution

1. Specify the level of significance α.
2. Determine whether the test is left-tailed, right-tailed, or two-tailed.
3. Find the critical value(s) z_0. When the hypothesis test is
   - **a.** *left-tailed*, find the z-score that corresponds to an area of α.
   - **b.** *right-tailed*, find the z-score that corresponds to an area of $1 - \alpha$.
   - **c.** *two-tailed*, find the z-scores that correspond to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s).

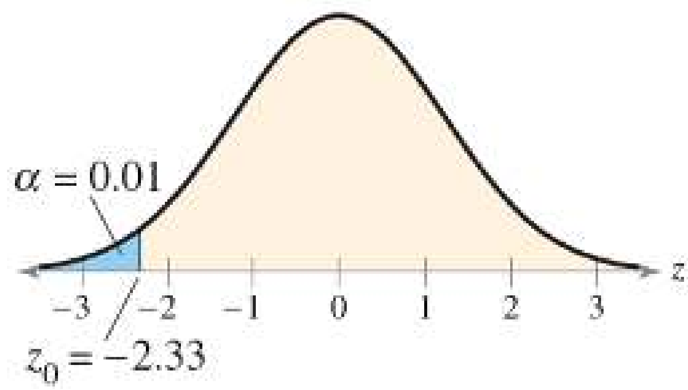
REJECTION REGIONS AND CRITICAL VALUES



Left-Tailed Test Right-Tailed Test Two-Tailed Test

Example Find the critical value and rejection region for a left-tailed test with $\alpha = 0.01$.

In Table 4, the z −score that is closest to an area of 0.01 is -2.33. So, the critical value is $z_0 = -2.33$. The rejection region is to the left of this critical value.



REJECTION REGIONS AND CRITICAL VALUES

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
− 3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
− 3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
− 3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
− 3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
− 3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
− 2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
− 2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
− 2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
− 2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
− 2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
− 2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
− 2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
− 2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
− 2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
− 2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
− 1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287

REJECTION REGIONS AND CRITICAL VALUES

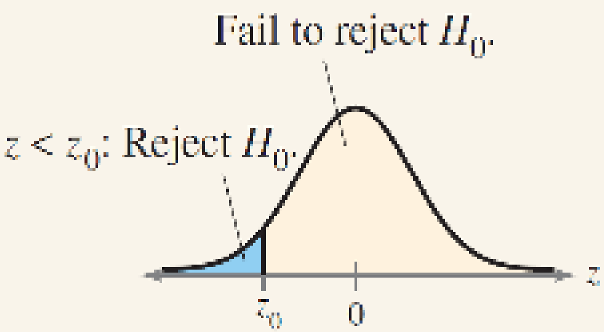
z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
−2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
−2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
−2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
−2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
−2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
−2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
−2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
−2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
−2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
−1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287

USING REJECTION REGIONS FOR A z –TEST

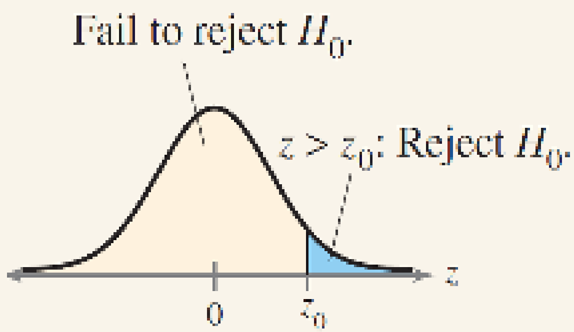
To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic z. If the standardized test statistic

1. is in the rejection region, then reject H_0.

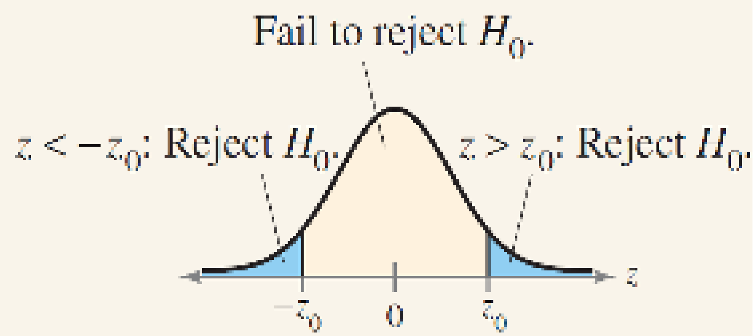
2. is *not* in the rejection region, then fail to reject H_0.



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

USING REJECTION REGIONS FOR A z –TEST

Using Rejection Regions for a z-Test for a Mean μ (σ Known)

IN WORDS	IN SYMBOLS
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.
5. Determine the rejection region(s).	

USING REJECTION REGIONS FOR A z –TEST

6. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

USING REJECTION REGIONS FOR A z −TEST

Example Employees at a construction and mining company claim that the mean salary of the company's mechanical engineers is less than that of one of its competitors, which is \$68,000. A random sample of 20 of the company's mechanical engineers has a mean salary of \$66,900. Assume the population standard deviation is \$5500 and the population is normally distributed. At $\alpha = 0.05$, test the employees' claim.

The claim is "the mean salary is less than \$68,000." So, the null and alternative hypotheses can be written as

$$H_0 \quad : \quad \mu \geq 68000$$
$$H_a \quad : \quad \mu < 68000 \quad \text{(Claim)}$$

USING REJECTION REGIONS FOR A z −TEST

Example Employees at a construction and mining company claim that the mean salary of the company's mechanical engineers is less than that of one of its competitors, which is \$68,000. A random sample of 20 of the company's mechanical engineers has a mean salary of \$66,900. Assume the population standard deviation is \$5500 and the population is normally distributed. At $\alpha = 0.05$, test the employees' claim.

$$H_0 \quad : \quad \mu \geq 68000$$

$$H_a \quad : \quad \mu < 68000 \quad (\text{Claim})$$

$\mu = 68000$

$\sigma = 5500$

$\bar{x} = 66900$

$n = 20$

$\alpha = 0.05$

USING REJECTION REGIONS FOR A z –TEST

Example $H_0 \quad : \quad \mu \geq 68000$

$H_a \quad : \quad \mu < 68000 \quad$ (Claim)

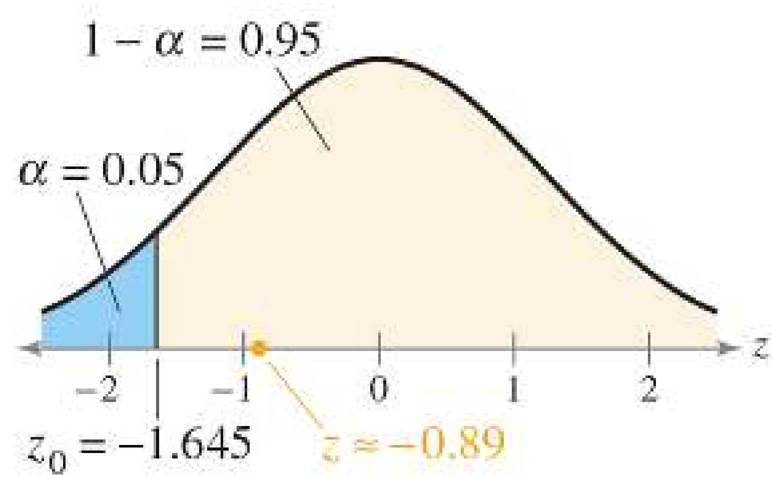
$\mu = 68000 \qquad n = 20$

$\sigma = 5500 \qquad \alpha = 0.05$

$\bar{x} = 66900$

Because the test is a left-tailed test and the level of significance is $\alpha = 0.05$, the critical value is $z_0 = -1.645$ and the rejection region is $z < -1.645$.

The standardized test statistic is



USING REJECTION REGIONS FOR A z −TEST

Example $H_0 \;:\; \mu \geq 68000$

$H_a \;:\; \mu < 68000$ (Claim)

$\mu = 68000$ $\quad n = 20$

$\sigma = 5500$ $\quad \alpha = 0.05$

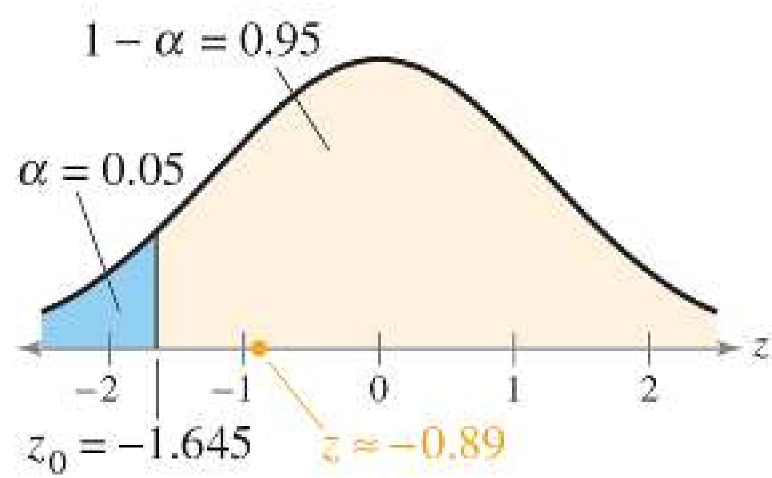
$\bar{x} = 66900$

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{66900 - 68000}{5500/\sqrt{20}} \approx -0.89$$

Because z is not in the rejection region, you fail to reject the null hypothesis.

There is not enough evidence at the 5% level of significance to support the employees' claim that the mean salary is less than $68,000.



USING REJECTION REGIONS FOR A z −TEST

Example A researcher claims that the mean annual cost of raising a child (age 2 and under) by husband-wife families in the U.S. is \$13,960. *In a random sample* of husband-wife families in the U.S., the mean annual cost of raising a child (age 2 and under) is \$13,725. The sample consists of 500 children. Assume the population standard deviation is \$2345. At $\alpha = 0.10$, is there enough evidence to reject the claim?

The claim is "the mean annual cost of raising a child (age 2 and under) by husband-wife families in the U.S. is \$13,960."

$$H_0 \quad : \quad \mu = 13960 \quad (\text{Claim})$$
$$H_a \quad : \quad \mu \neq 13960$$

USING REJECTION REGIONS FOR A z −TEST

Example A researcher claims that the mean annual cost of raising a child (age 2 and under) by husband-wife families in the U.S. is $13,960. *In a random sample* of husband-wife families in the U.S., the mean annual cost of raising a child (age 2 and under) is $13,725. The sample consists of 500 children. Assume the population standard deviation is $2345. At $\alpha = 0.10$, is there enough evidence to reject the claim?

$$\mu = 13960$$

$$\sigma = 2345$$

$$\bar{x} = 13725$$

$$n = 500$$

$$\alpha = 0.10$$

$H_0 \; : \; \mu = 13960$ (Claim)

$H_a \; : \; \mu \neq 13960$

USING REJECTION REGIONS FOR A z –TEST

Example $H_0 \quad : \quad \mu = 13960 \quad$ (Claim)

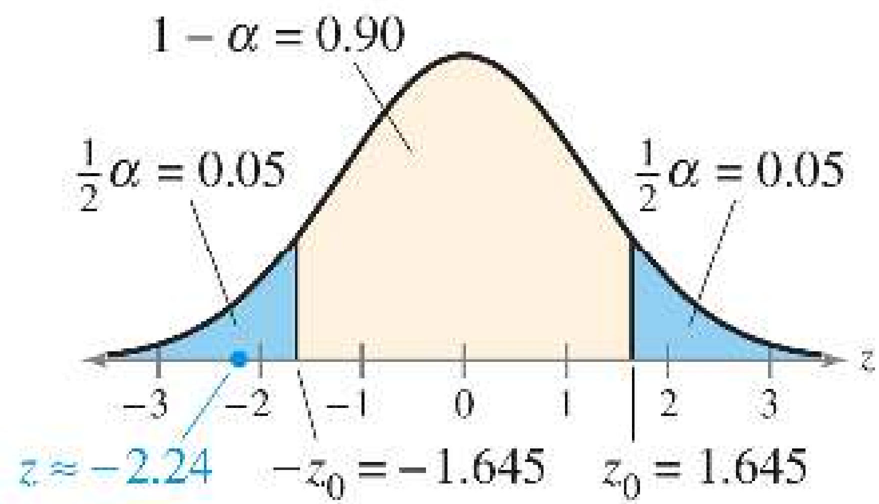
$H_a \quad : \quad \mu \neq 13960$

$\mu = 13960 \qquad n = 500$

$\sigma = 2345 \qquad \alpha = 0.10$

$\bar{x} = 13725$

Because the test is a two-tailed test and the level of significance is $\alpha = 0.10$, the critical values are $z_0 = -1.645$ and $z_0 = 1.645$. The rejection regions are $z < -1.645$ and $z > 1.645$.



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
− 3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
− 3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
− 3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
− 3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
− 3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
− 2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
− 2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
− 2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
− 2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
− 2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
− 2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
− 2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
− 2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
− 2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
− 2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
− 1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
− 1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
− 1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
− 1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
− 1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668

USING REJECTION REGIONS FOR A z –TEST

Example $H_0 \;:\; \mu = 13960$ (Claim)

$H_a \;:\; \mu \neq 13960$

$\mu = 13960$ $n = 500$

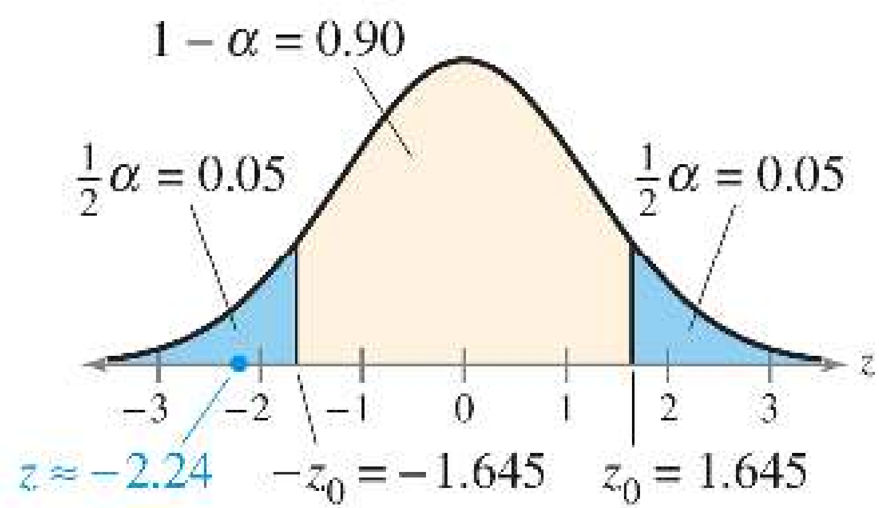
$\sigma = 2345$ $\alpha = 0.10$

$\bar{x} = 13725$

The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{13725 - 13960}{2345/\sqrt{500}} \approx -2.24$$

Because z is in the rejection region, you reject the null hypothesis.



USING REJECTION REGIONS FOR A z −TEST

Example

$H_0 \quad : \quad \mu = 13960 \quad$ (Claim)

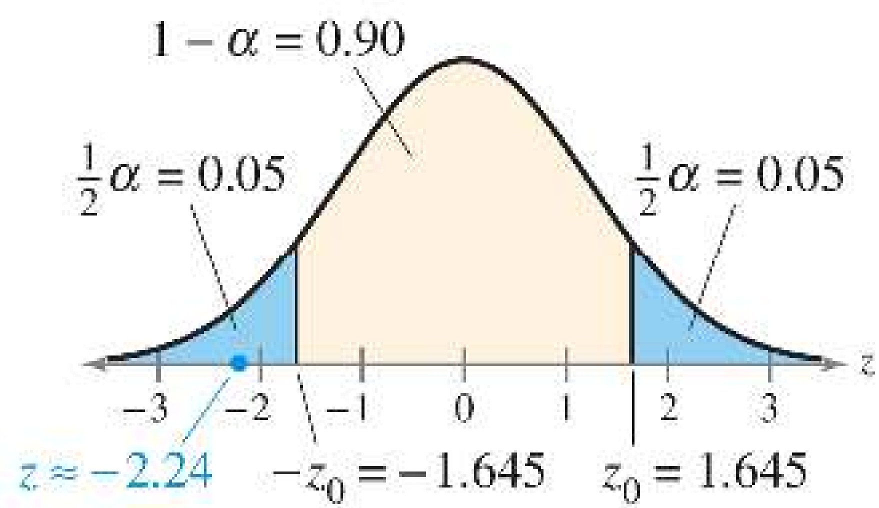
$H_a \quad : \quad \mu \neq 13960$

$\mu = 13960 \qquad n = 500$

$\sigma = 2345 \qquad \alpha = 0.10$

$\bar{x} = 13725$

There is enough evidence at the 10% level of significance to reject the claim that the mean annual cost of raising a child (age 2 and under) by husband-wife families in the U.S. is $13,960.



Course: Biostatistics

Lecture No: [14]

Chapter: [7]

Hypothesis Testing with One Sample

Section: [7.3]

Hypothesis Testing for the Mean (σ Unknown)

CRITICAL VALUES IN A t −DISTRIBUTION

- In many real-life situations, the population standard deviation is *not known*.
- When either the population has a normal distribution or the sample size is at least 30, you can still test the population mean μ.
- To do so, you can use the t −distribution with $n - 1$ degrees of freedom.

CRITICAL VALUES IN A t −DISTRIBUTION

Finding Critical Values in a *t*-Distribution

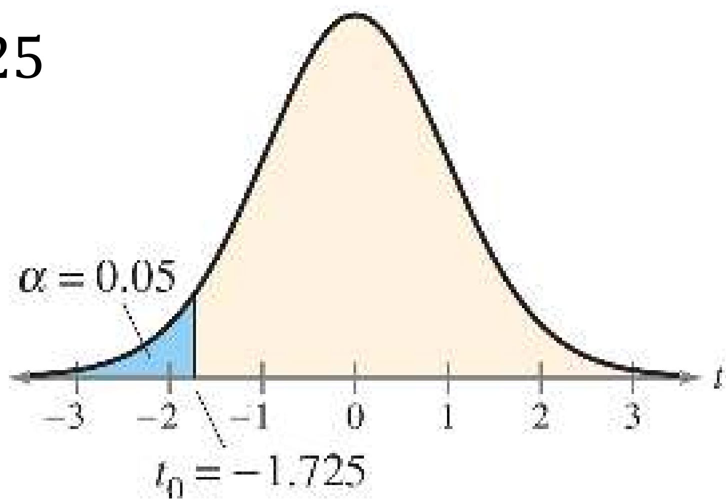
1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. $= n - 1$.
3. Find the critical value(s) using Table 5 in Appendix B in the row with $n - 1$ degrees of freedom. When the hypothesis test is
   a. *left-tailed,* use the "One Tail, α" column with a negative sign.
   b. *right-tailed,* use the "One Tail, α" column with a positive sign.
   c. *two-tailed,* use the "Two Tails, α" column with a negative and a positive sign.

CRITICAL VALUES IN A t −DISTRIBUTION

Example Find the critical value t_0 for a left-tailed test with $\alpha = 0.05$ and $n = 21$.

$$d.f. = n - 1 = 21 - 1 = 20$$

$$t_0 = -1.725$$



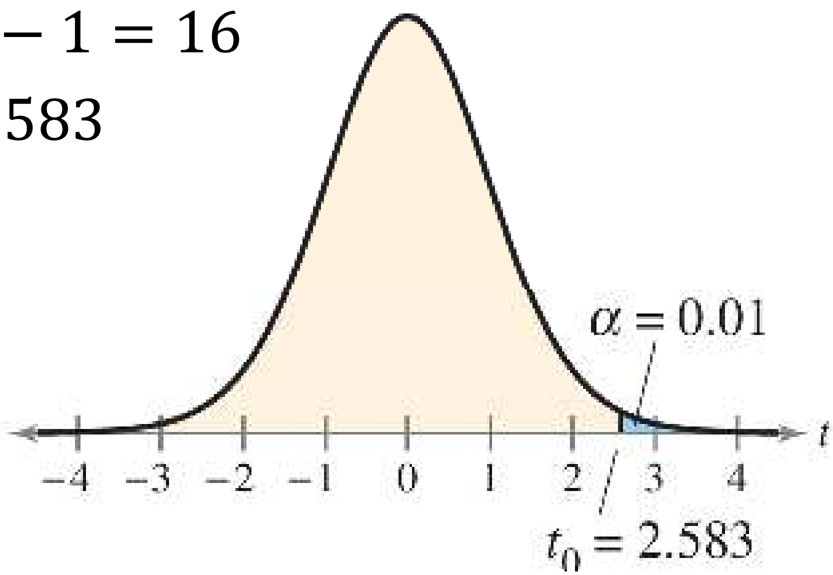
	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819

CRITICAL VALUES IN A t −DISTRIBUTION

Example Find the critical value t_0 for a right-tailed test with $\alpha = 0.01$ and $n = 17$.

$d.f. = n - 1 = 16$

$t_0 = 2.583$



d.f.	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
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16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819

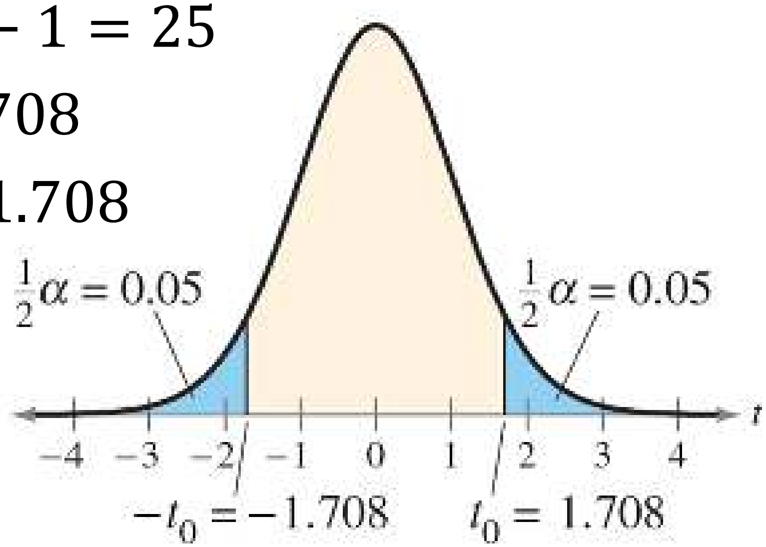
CRITICAL VALUES IN A t −DISTRIBUTION

Example Find the critical values $-t_0$ and t_0 for a two-tailed test with $\alpha = 0.10$ and $n = 26$.

$$d.f. = n - 1 = 25$$

$$t_0 = 1.708$$

$$-t_0 = -1.708$$



	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
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5		1.476	2.015	2.571	3.365	4.032
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7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
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14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807
24		1.318	1.711	2.064	2.492	2.797
25		1.316	1.708	2.060	2.485	2.787
26		1.315	1.706	2.056	2.479	2.779

THE t –TEST FOR A MEAN μ

The ***t*-test for a mean μ** is a statistical test for a population mean. The **test statistic** is the sample mean $\bar{x}$. The **standardized test statistic** is

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$ Standardized test statistic for μ (σ unknown)

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or $n \geq 30$.

The degrees of freedom are

$$\text{d.f.} = n - 1.$$

THE t –TEST FOR A MEAN μ

Using the t-Test for a Mean μ (σ Unknown)

IN WORDS	IN SYMBOLS
1. Verify that σ is not known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= n - 1$
5. Determine the critical value(s).	Use Table 5 in Appendix B.

THE t –TEST FOR A MEAN μ

6. Determine the rejection region(s).

7. Find the standardized test statistic and sketch the sampling distribution.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

8. Make a decision to reject or fail to reject the null hypothesis.

If t is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.

9. Interpret the decision in the context of the original claim.

THE t −TEST FOR A MEAN μ

Example A used car dealer says that the mean price of a two-year-old sedan (in good condition) is at least \$20,500. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of \$19,850 and a standard deviation of \$1084. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed.

$$\mu = 20500$$

$$H_0 \quad : \quad \mu \geq 20500 \quad (\text{Claim})$$

$$H_a \quad : \quad \mu < 20500$$

$$s = 1084$$

$$\bar{x} = 19850$$

$$n = 14$$

$$\alpha = 0.05$$

THE t –TEST FOR A MEAN μ

Example $H_0 \ : \ \mu \geq 20500$ (Claim)

$H_a \ : \ \mu < 20500$

$\mu = 20500$ $n = 14$

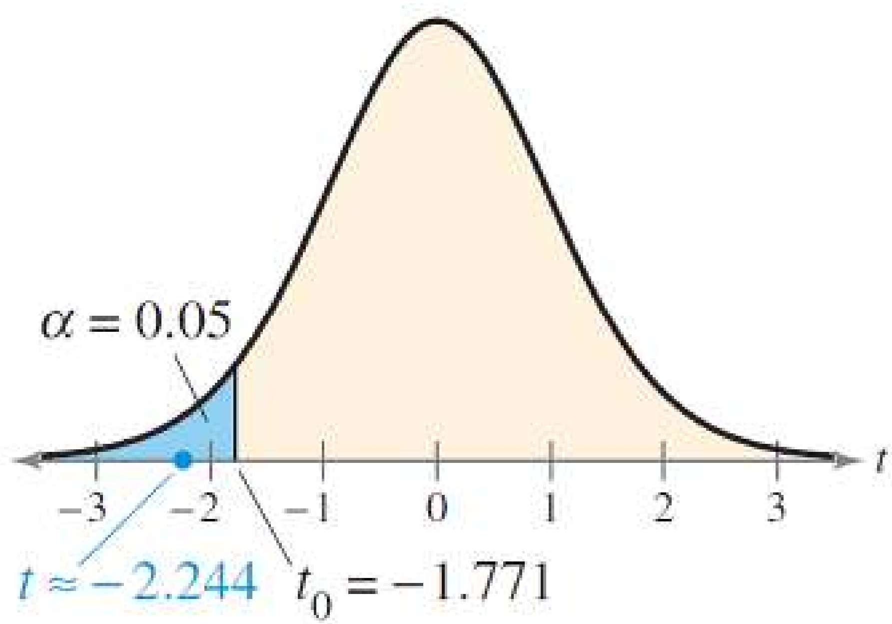
$s = 1084$ $\alpha = 0.05$

$\bar{x} = 19850$

The test is a left-tailed test

$d.f. = n - 1 = 13$

So, the critical value is $t_0 = -1.771$.



THE t –TEST FOR A MEAN μ

Example $H_0 \quad : \quad \mu \geq 20500$ (Claim)

$H_a \quad : \quad \mu < 20500$

$\mu = 20500 \qquad n = 14$

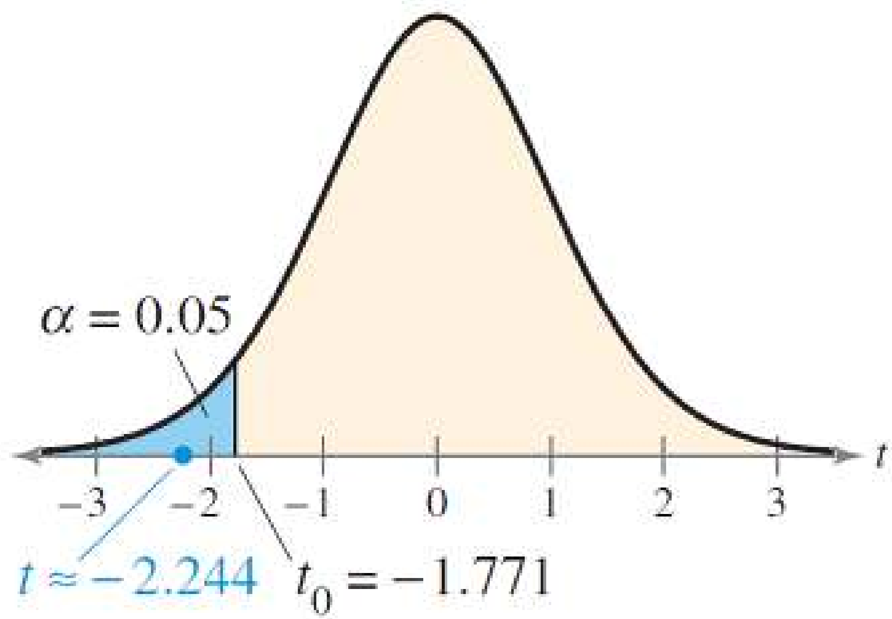
$s = 1084 \qquad \alpha = 0.05$

$\bar{x} = 19850$

The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{19850 - 20500}{1084/\sqrt{14}} \approx -2.244$$

Because t is in the rejection region, you reject the null hypothesis.



THE t –TEST FOR A MEAN μ

Example $H_0 \; : \; \mu \geq 20500$ (Claim)

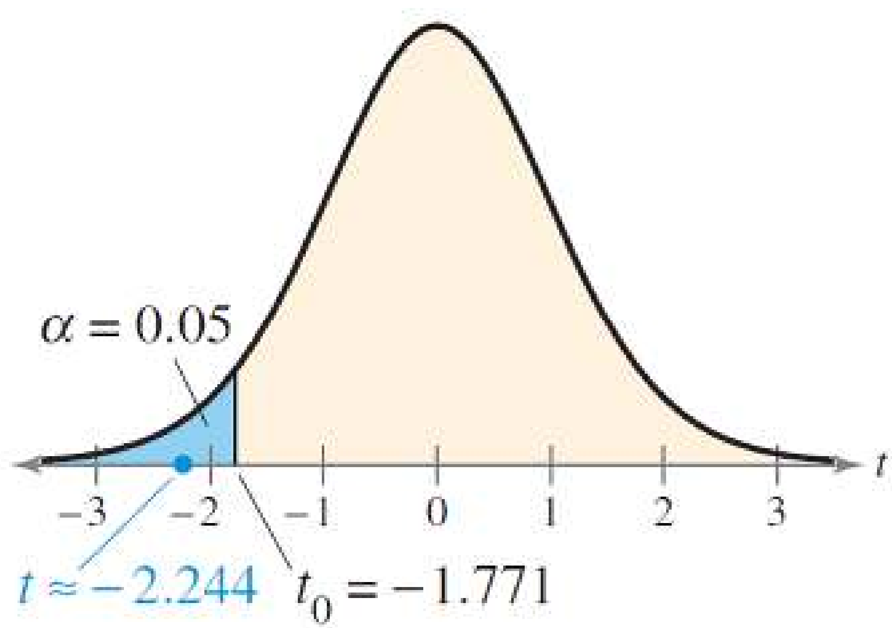
$H_a \; : \; \mu < 20500$

$\mu = 20500$ $n = 14$

$s = 1084$ $\alpha = 0.05$

$\bar{x} = 19850$

There is enough evidence at the 5% level of significance to reject the claim that the mean price of a two-year-old sedan is at least $20,500.



THE t −TEST FOR A MEAN μ

Example An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 39 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.35, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.05$?

$$H_0 \quad : \quad \mu = 6.8 \quad (\text{Claim})$$
$$H_a \quad : \quad \mu \neq 6.8$$

$\mu = 6.8$

$s = 0.35$

$\bar{x} = 6.7$

$n = 39$

$\alpha = 0.05$

THE t –TEST FOR A MEAN μ

Example H_0 : $\mu = 6.8$ (Claim)

H_a : $\mu \neq 6.8$

$\mu = 6.8$ $n = 39$

$s = 0.35$ $\alpha = 0.05$

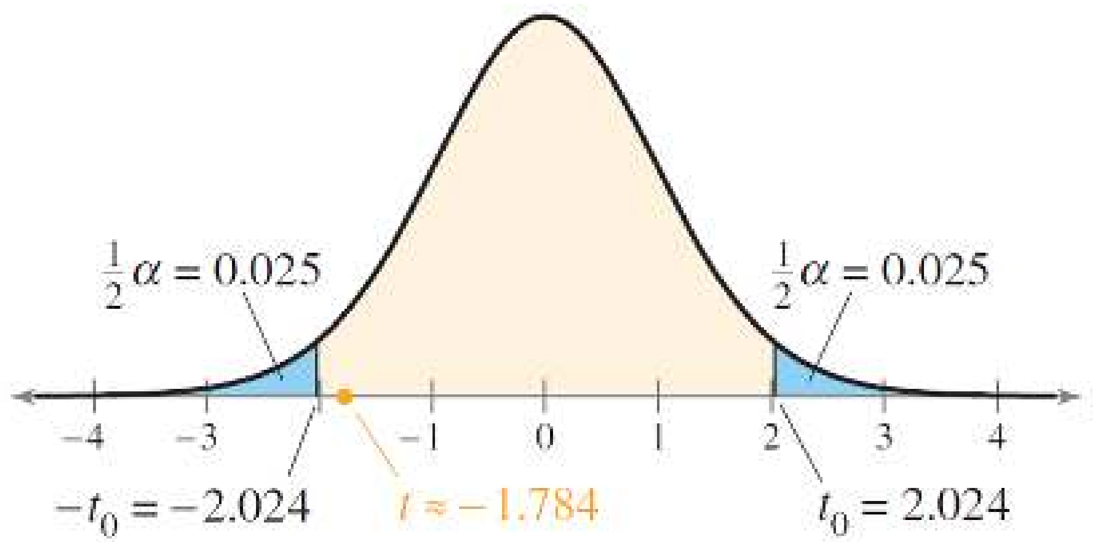
$\bar{x} = 6.7$

The test is a two-tailed test

$d.f. = n - 1 = 38$

So, the critical value are

$-t_0 = -2.024$ and $t_0 = 2.024$



THE t –TEST FOR A MEAN μ

Example $H_0 : \mu = 6.8$ (Claim)

$H_a : \mu \neq 6.8$

$\mu = 6.8$ $n = 39$

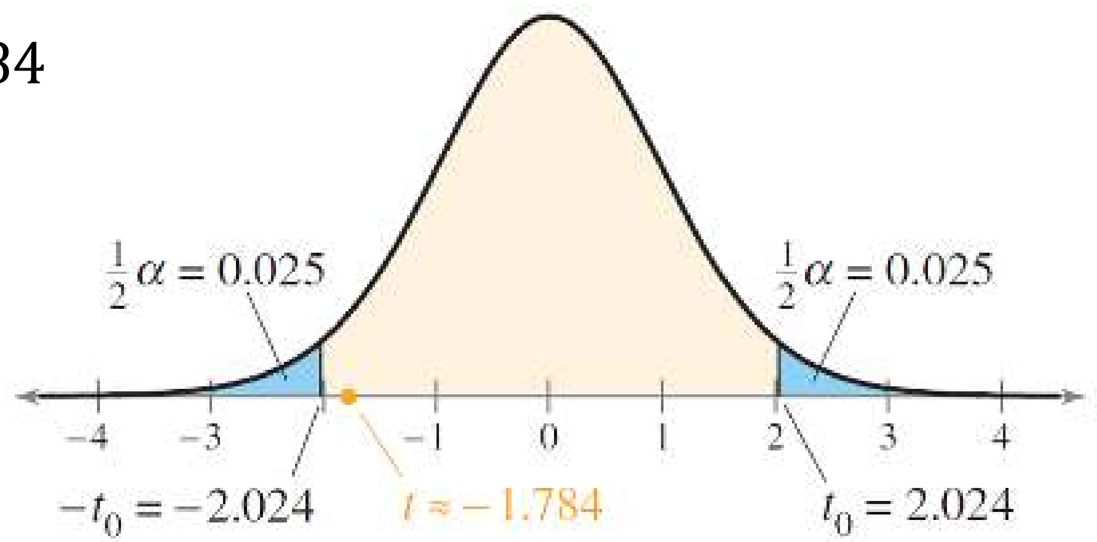
$s = 0.35$ $\alpha = 0.05$

$\bar{x} = 6.7$

The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{6.7 - 6.8}{0.35/\sqrt{39}} \approx -1.784$$

Because t is not in the rejection region, you fail to reject the null hypothesis.



THE t –TEST FOR A MEAN μ

Example $H_0 \;:\; \mu = 6.8$ (Claim)

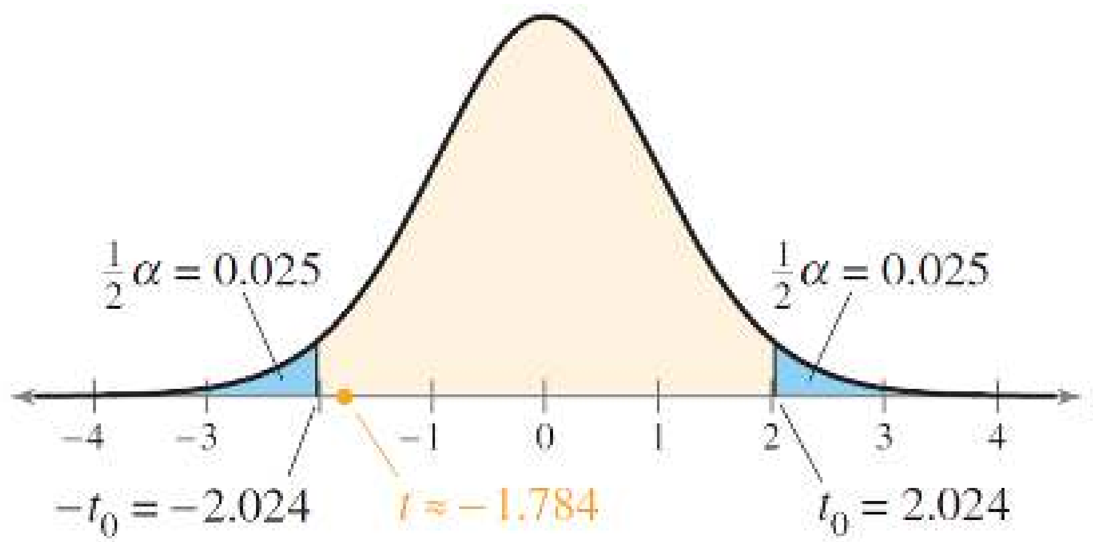
$H_a \;:\; \mu \neq 6.8$

$\mu = 6.8$ $\quad n = 39$

$s = 0.35$ $\quad \alpha = 0.05$

$\bar{x} = 6.7$

There is not enough evidence at the 5% level of significance to reject the claim that the mean pH level is 6.8.



Course: Biostatistics

Lecture No: [15]

Chapter: [7]

Hypothesis Testing with One Sample

Section: [7.4]

Hypothesis Testing for Proportions

HYPOTHESIS TEST FOR PROPORTIONS

- In this section, you will learn how to test a population proportion p.
- If $np \geq 5$ and $nq \geq 5$ for a binomial distribution, then the sampling distribution for $\hat{p}$ is approximately normal with a mean of $\mu_{\hat{p}} = p$ and a standard error of $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$.

HYPOTHESIS TEST FOR PROPORTIONS

- The z −test for a proportion p is a statistical test for a population proportion.
- The z −test **can be used when** a binomial distribution is given such that $np \geq 5$ and $nq \geq 5$.
- The **test statistic** is the sample proportion $\hat{p}$ and the **standardized test statistic is**

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

HYPOTHESIS TEST FOR PROPORTIONS

Using a z-Test for a Proportion p

IN WORDS	IN SYMBOLS
1. Verify that the sampling distribution of $\hat{p}$ can be approximated by a normal distribution.	$np \geq 5, nq \geq 5$
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.
5. Determine the rejection region(s).	

HYPOTHESIS TEST FOR PROPORTIONS

6. Find the standardized test statistic and sketch the sampling distribution.

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$

7. Make a decision to reject or fail to reject the null hypothesis.

If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.

8. Interpret the decision in the context of the original claim.

HYPOTHESIS TEST FOR PROPORTIONS

Example A researcher claims that less than 40% of U.S. cell phone owners use their phone for most of their online browsing. In a random sample of 100 adults, 31% say they use their phone for most of their online browsing. At $\alpha = 0.01$, is there enough evidence to support the researcher's claim?

$$H_0 \quad : \quad p \geq 0.40$$
$$H_a \quad : \quad p < 0.40 \quad (\text{Claim})$$

$$p = 0.40$$
$$q = 0.60$$
$$\hat{p} = 0.31$$
$$n = 100$$
$$\alpha = 0.01$$

$$np = (100)(0.40) = 40 \geq 5$$
$$nq = (100)(0.60) = 60 \geq 5$$

So, you can use a z −test.

HYPOTHESIS TEST FOR PROPORTIONS

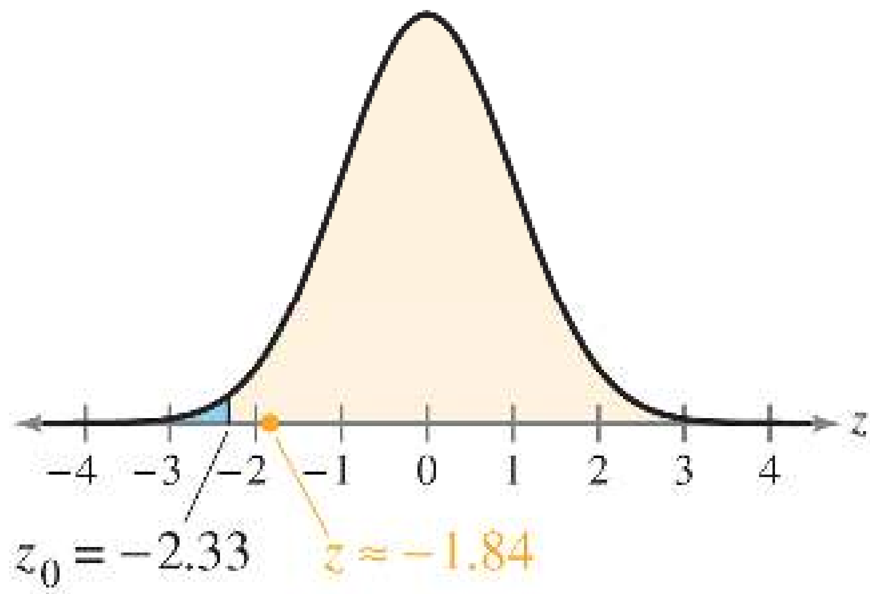
Example $H_0 \quad : \quad p \geq 0.40$

$H_a \quad : \quad p < 0.40 \quad$ (Claim)

$p = 0.40 \quad \hat{p} = 0.31 \quad \alpha = 0.01$

$q = 0.60 \quad n = 100$

Because the test is a left-tailed test and the level of significance is $\alpha = 0.01$, the critical value is $z_0 = -2.33$.



HYPOTHESIS TEST FOR PROPORTIONS

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
−2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
−2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
−2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
−2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
−2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
−2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
−2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
−2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
−2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228

HYPOTHESIS TEST FOR PROPORTIONS

Example $H_0 \; : \; p \geq 0.40$

$H_a \; : \; p < 0.40$ (Claim)

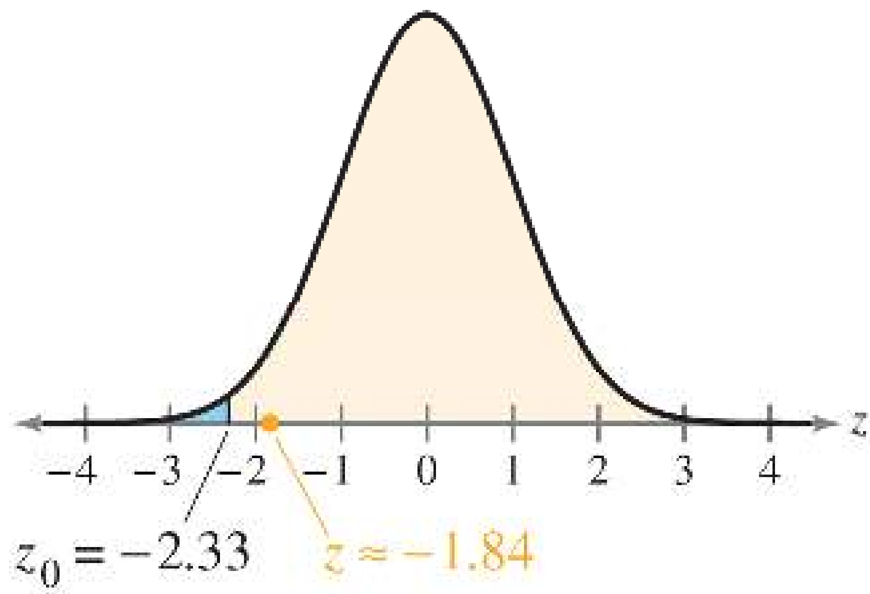
$p = 0.40$ $\hat{p} = 0.31$ $\alpha = 0.01$

$q = 0.60$ $n = 100$

Because the test is a left-tailed test and the level of significance is $\alpha = 0.01$, the critical value is $z_0 = -2.33$.

The standardized test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.31 - 0.4}{\sqrt{(0.4)(0.6)/100}} \approx -1.84$$



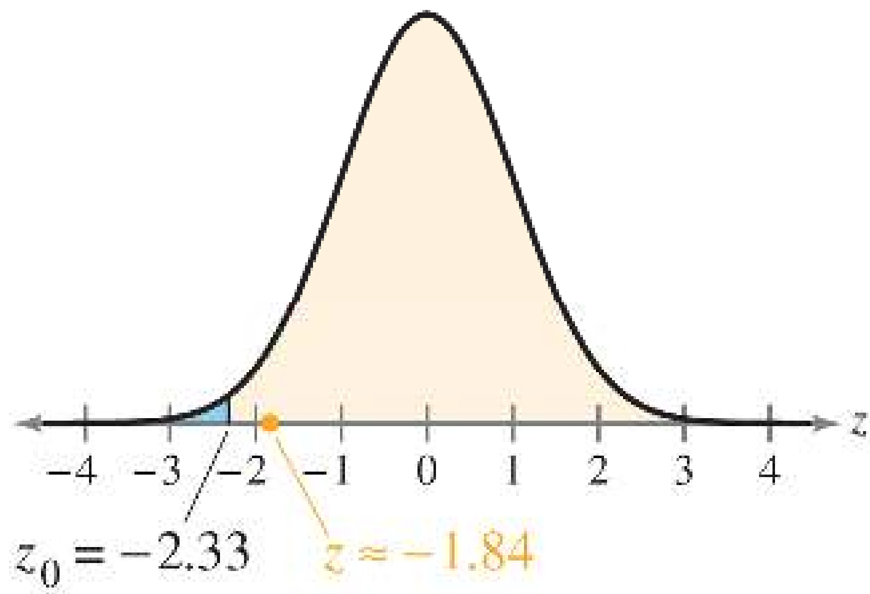
HYPOTHESIS TEST FOR PROPORTIONS

Example $H_0 \quad : \quad p \geq 0.40$ $\qquad p = 0.40 \qquad \hat{p} = 0.31 \qquad \alpha = 0.01$

$H_a \quad : \quad p < 0.40 \quad (\text{Claim})$ $\qquad q = 0.60 \qquad n = 100$

Because z is not in the rejection region, you fail to reject the null hypothesis.

There is not enough evidence at the 1% level of significance to support the claim that less than 40% of U.S. cell phone owners use their phone for most of their online browsing.



HYPOTHESIS TEST FOR PROPORTIONS

Example A researcher claims that 86% of college graduates say their college degree has been a good investment. In a random sample of 1000 graduates, 845 say their college degree has been a good investment. At $\alpha = 0.10$, is there enough evidence to reject the researcher's claim?

$$H_0 \ : \ p = 0.86 \quad (\text{Claim})$$
$$H_a \ : \ p \neq 0.86$$

$$np = (1000)(0.86) = 860 \geq 5$$
$$nq = (1000)(0.14) = 140 \geq 5$$

So, you can use a z −test.

$$p = 0.86$$
$$q = 0.14$$
$$\hat{p} = \frac{845}{1000} = 0.845$$
$$n = 1000$$
$$\alpha = 0.10$$

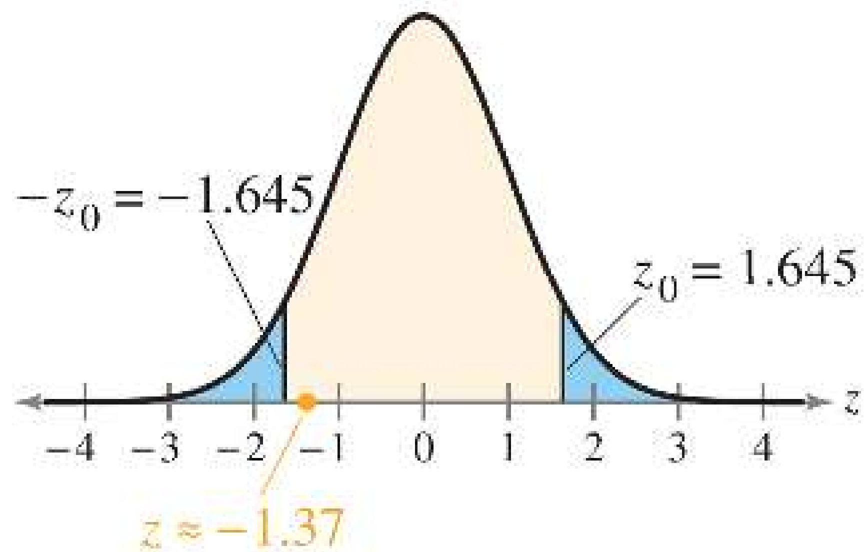
HYPOTHESIS TEST FOR PROPORTIONS

Example $H_0 \quad : \quad p = 0.86 \quad$ (Claim) $\qquad p = 0.86 \qquad n = 1000$

$H_a \quad : \quad p \neq 0.86 \qquad q = 0.14 \qquad \alpha = 0.10$

$$\hat{p} = \frac{845}{1000} = 0.845$$

Because the test is a two-tailed test and the level of significance is $\alpha = 0.10$, the critical values are $-z_0 = -1.645$ and $z_0 = 1.645$.



The standardized test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.845 - 0.86}{\sqrt{(0.86)(0.14)/1000}}$$
$$\approx -1.37$$

HYPOTHESIS TEST FOR PROPORTIONS

Example $H_0 \quad : \quad p = 0.86 \quad$ (Claim)

$H_a \quad : \quad p \neq 0.86$

$p = 0.86$

$q = 0.14$

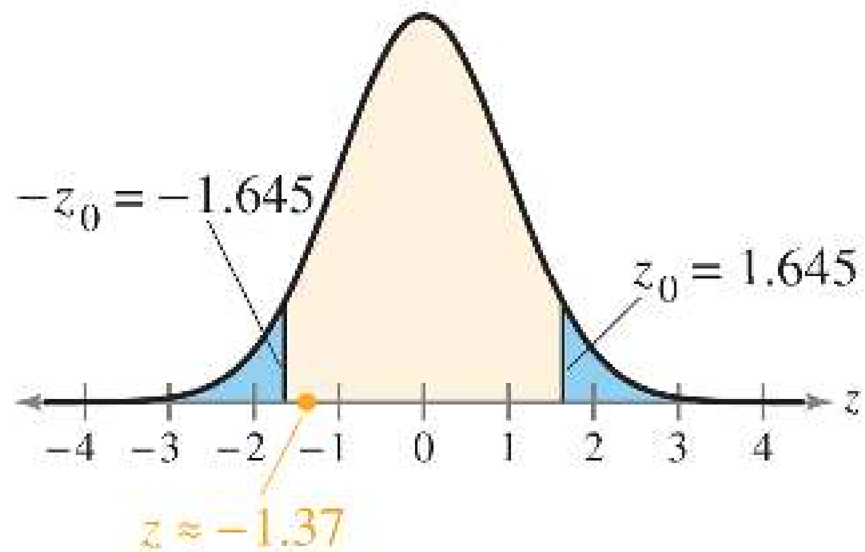
$n = 1000$

$\alpha = 0.10$

$$\hat{p} = \frac{845}{1000} = 0.845$$

Because z is not in the rejection region, you fail to reject the null hypothesis.

There is not enough evidence at the 10% level of significance to reject the claim that 86% of college graduates say their college degree has been a good investment.



Course: Biostatistics

Lecture No: [15]

Chapter: [7]

Hypothesis Testing with One Sample

Section: [7.5]

Hypothesis Testing for Variance and Standard Deviation

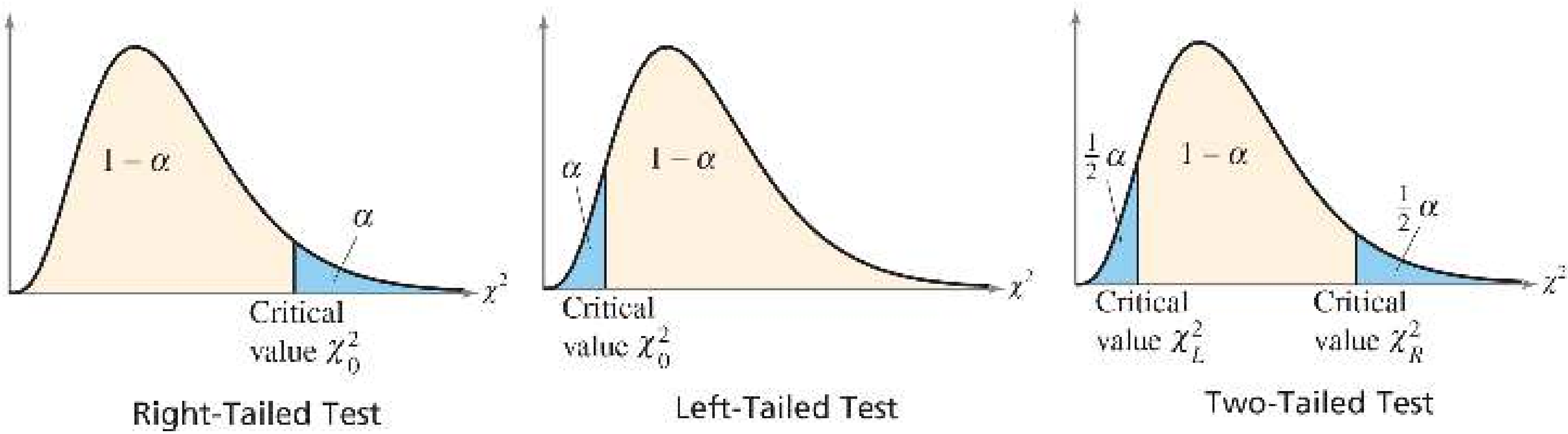
CRITICAL VALUES FOR A CHI-SQUARE TEST

For a normally distributed population, you can test the variance and standard deviation of the process using the chi-square distribution with $n - 1$ degrees of freedom.

Finding Critical Values for a Chi-Square Test

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. $= n - 1$.
3. The critical values for the chi-square distribution are found in Table 6 in Appendix B. To find the critical value(s) for a
   - **a.** *right-tailed test,* use the value that corresponds to d.f. and α.
   - **b.** *left-tailed test,* use the value that corresponds to d.f. and $1 - \alpha$.
   - **c.** *two-tailed test,* use the values that correspond to d.f. and $\frac{1}{2}\alpha$, and d.f. and $1 - \frac{1}{2}\alpha$.

CRITICAL VALUES FOR A CHI-SQUARE TEST

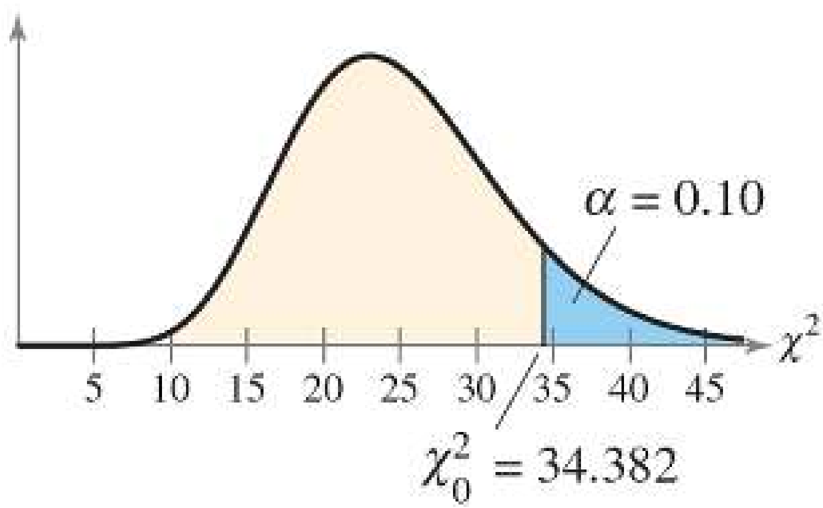


CRITICAL VALUES FOR A CHI-SQUARE TEST

Example

Find the critical value χ_0^2 for a right-tailed test when $n = 26$ and $\alpha = 0.10$.

$\text{d.f.} = n - 1 = 25$ $\qquad$ $\chi_0^2 = 34.382$



Degrees of freedom	α					
	0.995	0.99	0.975	0.95	0.90	0.10
1	—	—	0.001	0.004	0.016	2.706
2	0.010	0.020	0.051	0.103	0.211	4.605
3	0.072	0.115	0.216	0.352	0.584	6.251
4	0.207	0.297	0.484	0.711	1.064	7.779
5	0.412	0.554	0.831	1.145	1.610	9.236
6	0.676	0.872	1.237	1.635	2.204	10.645
7	0.989	1.239	1.690	2.167	2.833	12.017
8	1.344	1.646	2.180	2.733	3.490	13.362
9	1.735	2.088	2.700	3.325	4.168	14.684
10	2.156	2.558	3.247	3.940	4.865	15.987
11	2.603	3.053	3.816	4.575	5.578	17.275
12	3.074	3.571	4.404	5.226	6.304	18.549
13	3.565	4.107	5.009	5.892	7.042	19.812
14	4.075	4.660	5.629	6.571	7.790	21.064
15	4.601	5.229	6.262	7.261	8.547	22.307
16	5.142	5.812	6.908	7.962	9.312	23.542
17	5.697	6.408	7.564	8.672	10.085	24.769
18	6.265	7.015	8.231	9.390	10.865	25.989
19	6.844	7.633	8.907	10.117	11.651	27.204
20	7.434	8.260	9.591	10.851	12.443	28.412
21	8.034	8.897	10.283	11.591	13.240	29.615
22	8.643	9.542	10.982	12.338	14.042	30.813
23	9.260	10.196	11.689	13.091	14.848	32.007
24	9.886	10.856	12.401	13.848	15.659	33.196
25	10.520	11.524	13.120	14.611	16.473	34.382

CRITICAL VALUES FOR A CHI-SQUARE TEST

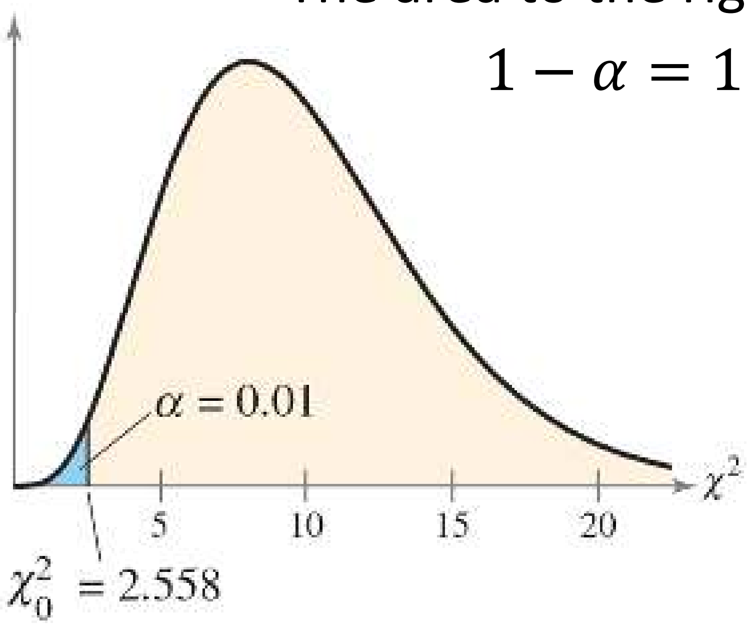
Example Find the critical value χ_0^2 for a left-tailed test when $n = 11$ and $\alpha = 0.01$.

$$\text{d.f.} = n - 1 = 10$$

The area to the right of the critical value is

$$1 - \alpha = 1 - 0.01 = 0.99$$

$$\chi_0^2 = 2.558$$



Degrees of freedom	0.995	0.99
1	—	—
2	0.010	0.020
3	0.072	0.115
4	0.207	0.297
5	0.412	0.554
6	0.676	0.872
7	0.989	1.239
8	1.344	1.646
9	1.735	2.088
10	2.156	2.558
11	2.603	3.053

CRITICAL VALUES FOR A CHI-SQUARE TEST

Example Find the critical values χ_L^2 and χ_R^2 for a two-tailed test when $n = 9$ and $\alpha = 0.05$.

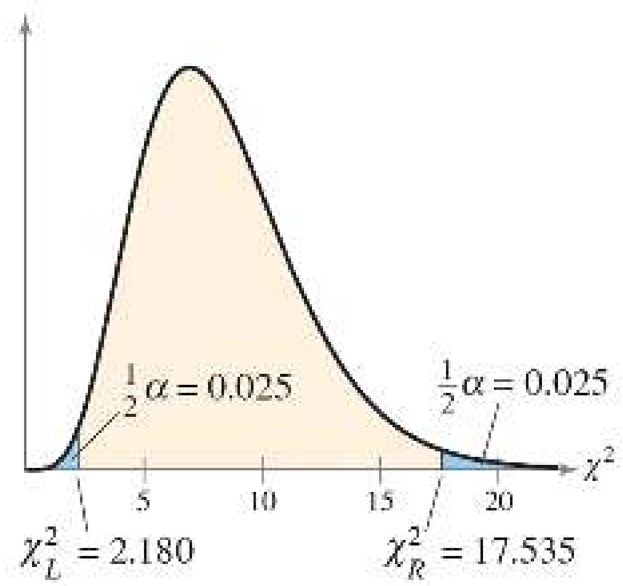
$$\text{d.f.} = n - 1 = 8$$

The areas to the right of the critical values are

$$\frac{1}{2}\alpha = \frac{0.05}{2} = 0.025$$

$$1 - \frac{1}{2}\alpha = 1 - 0.025 = 0.975$$

$$\chi_L^2 = 2.180 \qquad \chi_R^2 = 17.535$$



THE CHI-SQUARE TEST

- To test a variance σ^2 or a standard deviation σ of a population that is normally distributed, you can use the chi-square test.
- The chi-square test for a variance or standard deviation is *not* as robust (متانة) as the tests for the population mean μ or the population proportion p.
- So, it is essential in performing a chi-square test for a variance or standard deviation that the population be normally distributed.
- The results can be misleading (مضلل) when the population is not normal.

THE CHI-SQUARE TEST

CHI-SQUARE TEST FOR A VARIANCE σ^2 OR STANDARD DEVIATION σ

The **chi-square test for a variance σ^2 or standard deviation σ** is a statistical test for a population variance or standard deviation. The chi-square test can only be used when the population is normal. The **test statistic** is s^2 and the **standardized test statistic**

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Standardized test statistic for σ^2 or σ

follows a chi-square distribution with degrees of freedom

$$\text{d.f.} = n - 1.$$

THE CHI-SQUARE TEST

Using the Chi-Square Test for a Variance σ^2 or a Standard Deviation σ

IN WORDS	IN SYMBOLS
1. Verify that the sample is random and the population is normally distributed.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= n - 1$
5. Determine the critical value(s).	Use Table 6 in Appendix B.

THE CHI-SQUARE TEST

6. Determine the rejection region(s).

7. Find the standardized test statistic and sketch the sampling distribution.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

8. Make a decision to reject or fail to reject the null hypothesis.

If χ^2 is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.

9. Interpret the decision in the context of the original claim.

THE CHI-SQUARE TEST

Example A dairy processing company (شركة ألبان) claims that the variance of the amount of fat in the whole milk (حليب كامل الدسم) processed by the company is no more than 0.25. You suspect this is wrong and find that a random sample of 41 milk containers has a variance of 0.27. At $\alpha = 0.05$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

$$H_0 \quad : \quad \sigma^2 \leq 0.25 \quad (\text{Claim})$$
$$H_a \quad : \quad \sigma^2 > 0.25$$

$$\text{d.f.} = n - 1 = 41 - 1 = 40$$

The test is a right-tailed test

$$s^2 = 0.27 \qquad \alpha = 0.05$$

THE CHI-SQUARE TEST

Example $H_0 \quad : \quad \sigma^2 \leq 0.25 \quad$ (Claim)

$H_a \quad : \quad \sigma^2 > 0.25$

The test is a right-tailed test

$\text{d.f.} = n - 1 = 41 - 1 = 40$

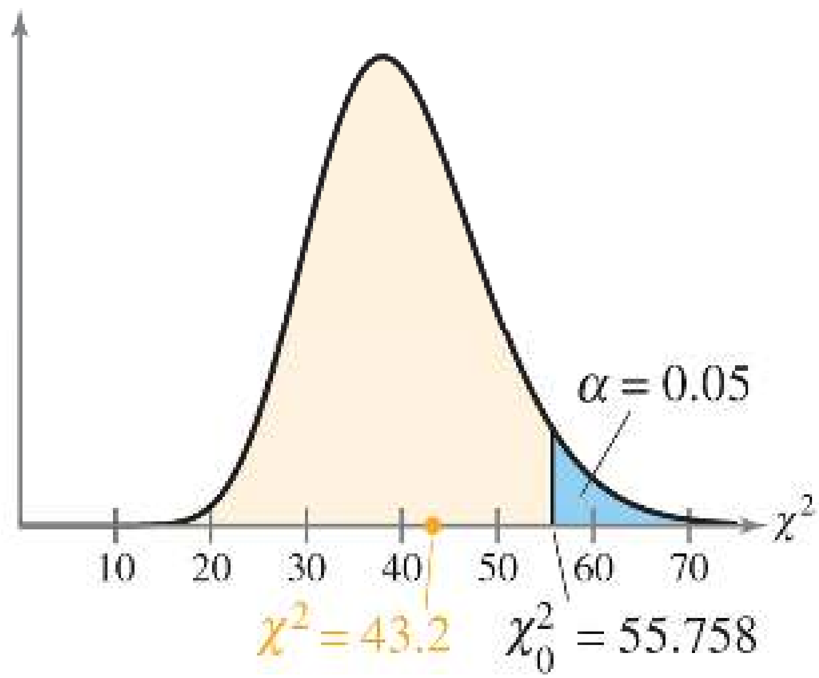
$s^2 = 0.27 \qquad \alpha = 0.05$

So, the critical value is $\chi_0^2 = 55.758$

The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(40)(0.27)}{0.25} = 43.2$$

Because χ^2 is not in the rejection region, you fail to reject the null hypothesis.



THE CHI-SQUARE TEST

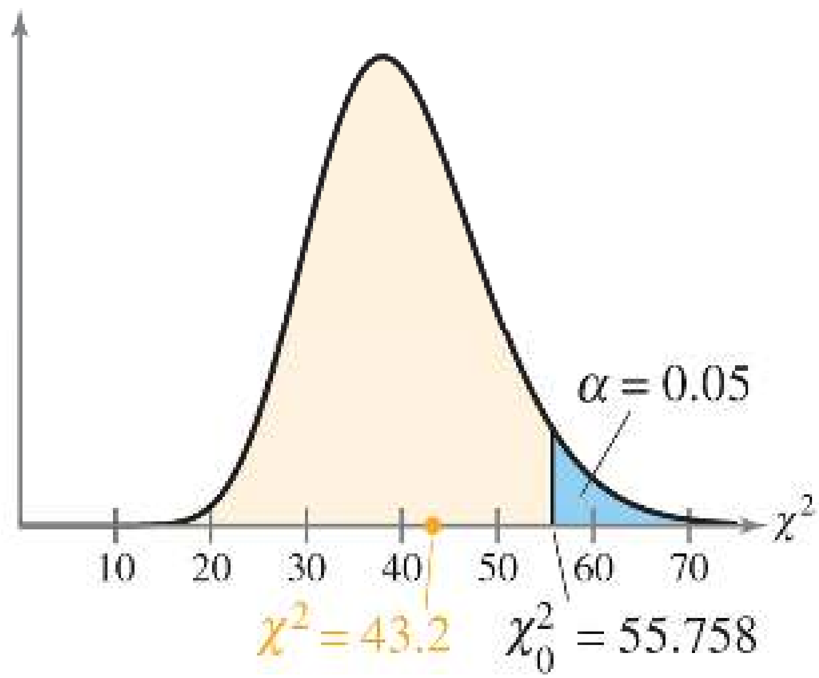
Example $H_0 \quad : \quad \sigma^2 \leq 0.25 \quad$ (Claim)

$H_a \quad : \quad \sigma^2 > 0.25$

The test is a right-tailed test

$\text{d.f.} = n - 1 = 41 - 1 = 40$

$s^2 = 0.27 \qquad \alpha = 0.05$



There is not enough evidence at the 5% level of significance to reject the company's claim that the variance of the amount of fat in the whole milk is no more than 0.25.

THE CHI-SQUARE TEST

Example A company claims that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes. A random sample of 25 incoming telephone calls has a standard deviation of 1.1 minutes. At $\alpha = 0.10$, is there enough evidence to support the company's claim? Assume the population is normally distributed.

$H_0 \quad : \quad \sigma \geq 1.4$

$H_a \quad : \quad \sigma < 1.4 \quad$ (Claim)

$s = 1.1 \qquad \alpha = 0.10$

$\text{d.f.} = n - 1 = 25 - 1 = 24$

The test is a left-tailed test

THE CHI-SQUARE TEST

Example $H_0 \;:\; \sigma \geq 1.4$

$H_a \;:\; \sigma < 1.4$ (Claim)

The test is a left-tailed test

$s = 1.1$ $\alpha = 0.10$

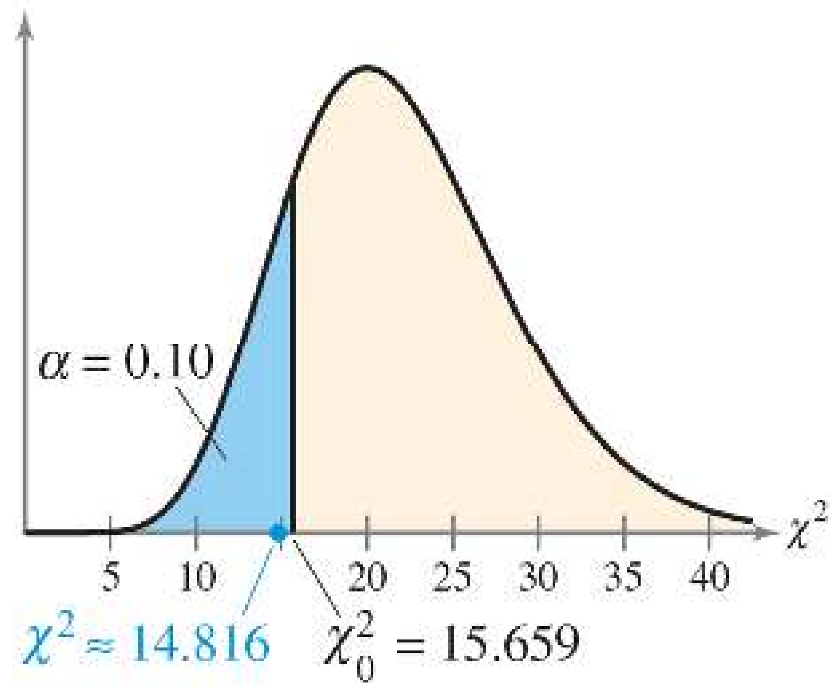
$\text{d.f.} = n - 1 = 25 - 1 = 24$

So, the critical value with $\text{d.f.} = 24$ and $1 - \alpha = 0.90$ is $\chi_0^2 = 15.659$

The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24)(1.1^2)}{1.4^2} \approx 14.816$$

Because χ^2 is in the rejection region, you reject the null hypothesis.



THE CHI-SQUARE TEST

Example $H_0 \ : \ \sigma \geq 1.4$

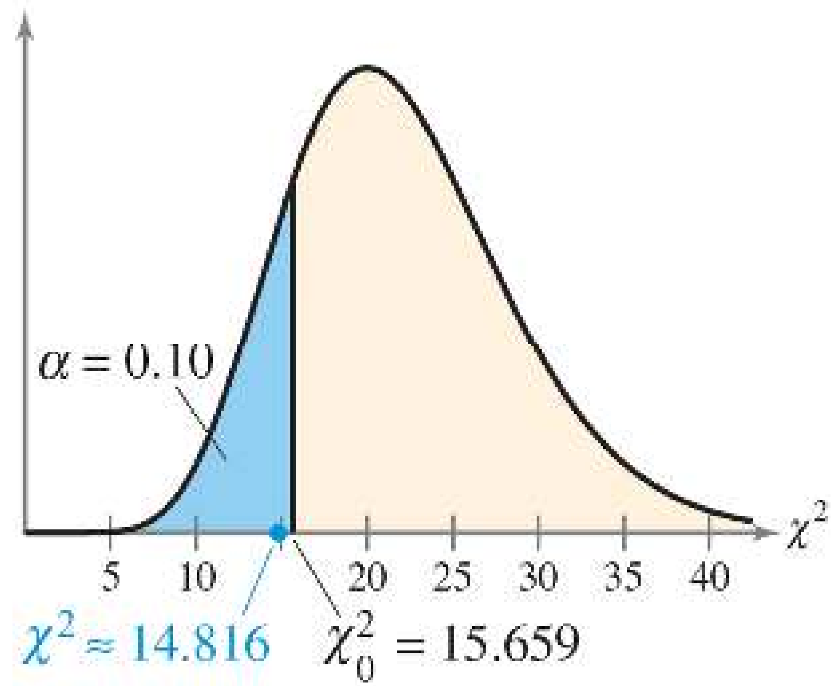
$H_a \ : \ \sigma < 1.4 \quad$ (Claim)

The test is a left-tailed test

$s = 1.1 \qquad \alpha = 0.05$

$\text{d.f.} = n - 1 = 25 - 1 = 24$

There is enough evidence at the 10% level of significance to support the claim that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes.



THE CHI-SQUARE TEST

Example A sporting goods manufacturer claims that the variance of the strengths of a certain fishing line is 15.9. A random sample of 15 fishing line spools has a variance of 21.8. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim? Assume the population is normally distributed.

$H_0 \;:\; \sigma^2 = 15.9$ (Claim) $\qquad s^2 = 21.8 \qquad \alpha = 0.05$

$H_a \;:\; \sigma^2 \neq 15.9 \qquad \text{d.f.} = n - 1 = 15 - 1 = 14$

The test is a two-tailed test

THE CHI-SQUARE TEST

Example $H_0 : \sigma^2 = 15.9$ (Claim)

$H_a : \sigma^2 \neq 15.9$

The test is a two-tailed test

$s^2 = 21.8 \quad \alpha = 0.05$

$\text{d.f.} = n - 1 = 15 - 1 = 14$

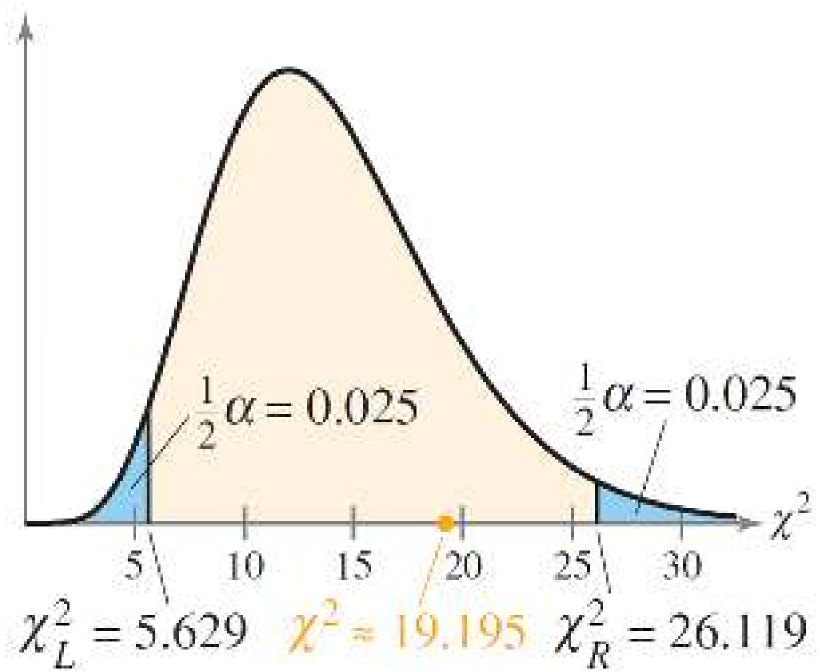
So, the critical values are $\chi_L^2 = 5.629$

$\chi_R^2 = 26.119$

The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(14)(21.8)}{15.9} \approx 19.195$$

Because χ^2 is not in the rejection region, you fail to reject the null hypothesis.



THE CHI-SQUARE TEST

Example $H_0 \;:\; \sigma^2 = 15.9$ (Claim)

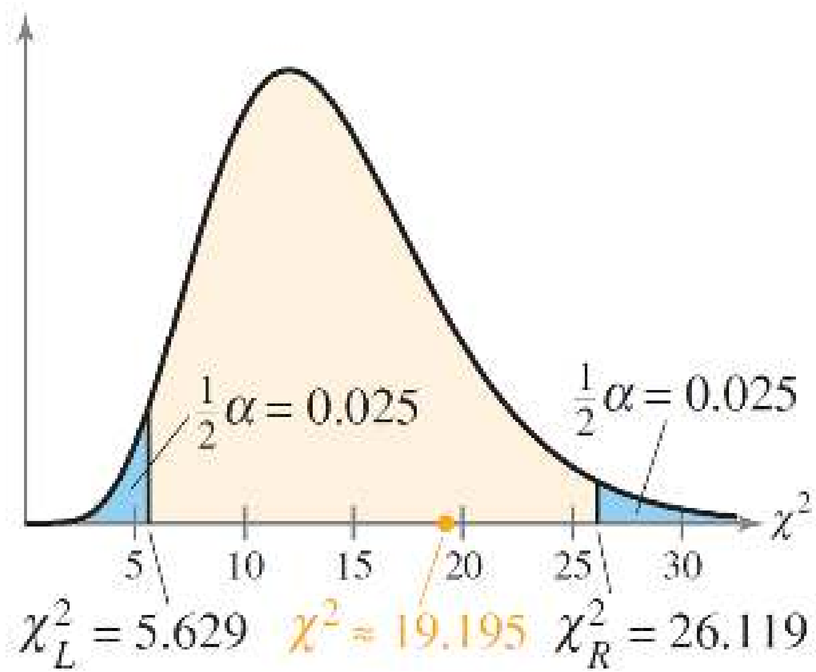
$H_a \;:\; \sigma^2 \neq 15.9$

The test is a two-tailed test

$s^2 = 21.8 \qquad \alpha = 0.05$

$\text{d.f.} = n - 1 = 15 - 1 = 14$

There is not enough evidence at the 5% level of significance to reject the claim that the variance of the strengths of the fishing line is 15.9.



Course: Biostatistics

Lecture No: [17]

Chapter: [8]

Hypothesis Testing with Two Sample

Section: [8.1]

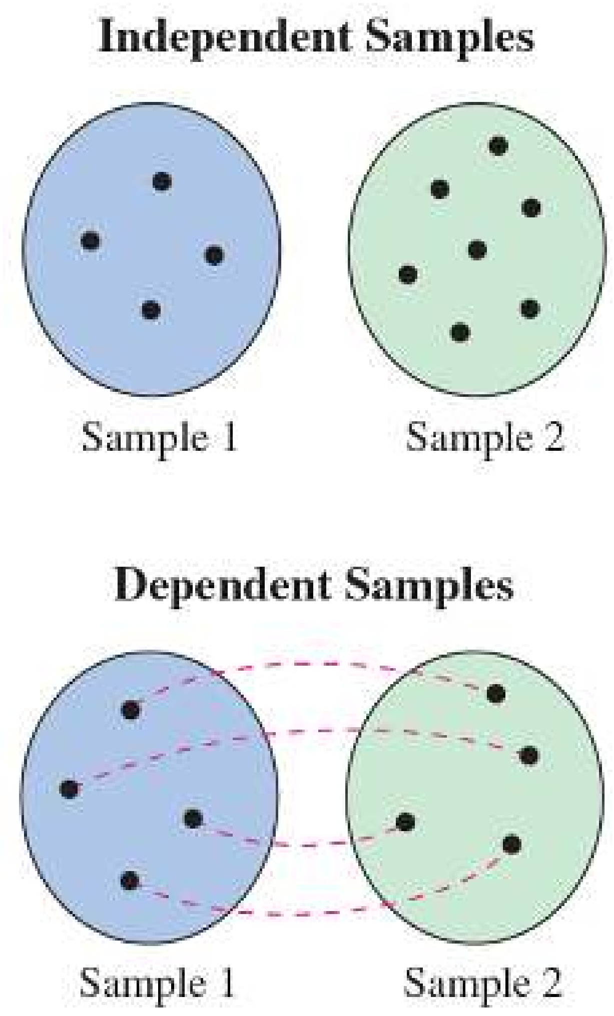
Testing the Difference Between Means (Independent Samples, σ_1 and σ_2 Known)

WHERE YOU'VE BEEN AND WHERE YOU'RE GOING

- In Chapter 7, you learned *how to test a claim about a population parameter*, basing your decision on sample statistics and their sampling distributions.
- In this chapter, you will continue your study of inferential statistics and hypothesis testing.
- But instead of testing a hypothesis about a single population, you will learn how to test a hypothesis that compares two populations.

INDEPENDENT AND DEPENDENT SAMPLES

- Two samples are **independent** when *the sample selected from one population is not related to the sample selected from the second population*.
- Two samples are **dependent** when *each member of one sample corresponds to a member of the other sample*.
- Dependent samples are also called **paired samples** or **matched samples**.



INDEPENDENT AND DEPENDENT SAMPLES

Example Classify each pair of samples as independent or dependent and justify your answer.

Sample [1]
Weights of 65 college students before their freshman year begins

Sample [2]
Weights of the same 65 college students after their freshman year

- These samples are dependent.
- Because the weights of the same students are taken, the samples are related.
- The samples can be paired with respect to each student.

INDEPENDENT AND DEPENDENT SAMPLES

Example Classify each pair of samples as independent or dependent and justify your answer.

Sample [1]
Scores for 38 adult males on a psychological screening test for attention-deficit hyperactivity disorder

Sample [2]
Scores for 50 adult females on a psychological screening test for attention-deficit hyperactivity disorder

- These samples are independent.
- It is not possible to form a pairing between the members of samples, the sample sizes are different, and the data represent scores for different individuals.

AN OVERVIEW OF TWO-SAMPLE HYPOTHESIS TESTING

In this section, you will learn how to test a claim comparing the means of two different populations using independent samples.

DEFINITION

For a two-sample hypothesis test with independent samples,

1. the **null hypothesis** H_0 is a statistical hypothesis that usually states there is no difference between the parameters of two populations. The null hypothesis always contains the symbol $\leq$, $=$, or $\geq$.
2. the **alternative hypothesis** H_a is a statistical hypothesis that is true when H_0 is false. The alternative hypothesis contains the symbol $>$, $\neq$, or $<$.

AN OVERVIEW OF TWO-SAMPLE HYPOTHESIS TESTING

- To write the null and alternative hypotheses for a two-sample hypothesis test with independent samples, translate the claim made about the population parameters from a verbal statement to a mathematical statement.
- Then, write its complementary statement.

$H_0: \mu_1 = \mu_2$	$H_0: \mu_1 \leq \mu_2$	$H_0: \mu_1 \geq \mu_2$
$H_a: \mu_1 \neq \mu_2$	$H_a: \mu_1 > \mu_2$	$H_a: \mu_1 < \mu_2$

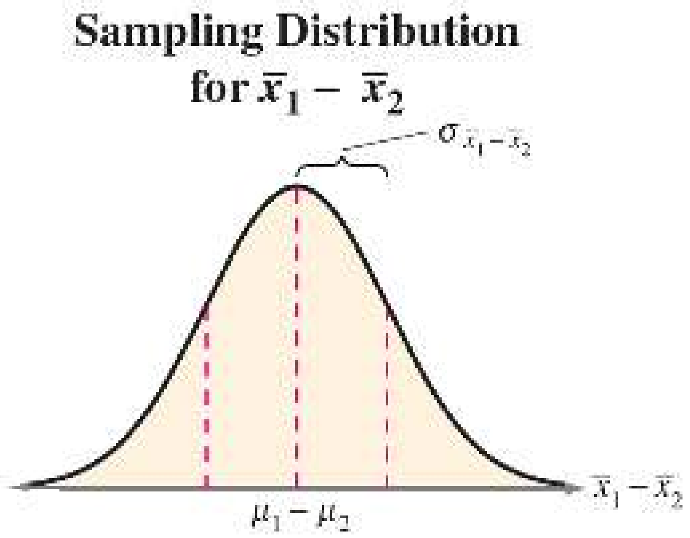
- Regardless of which hypotheses you use, you always assume there is no difference between the population means $(\mu_1 = \mu_2)$.

TWO-SAMPLE z −TEST FOR THE DIFFERENCE BETWEEN MEANS

- In the remainder of this section, you will learn how to perform a *z −test for the difference between two population means μ_1 and μ_2 when the samples are independent.*
- These conditions are *necessary* to perform such a test.
  - ✓ The population standard deviations are known.
  - ✓ The samples are randomly selected.
  - ✓ The samples are independent.
  - ✓ The populations are normally distributed, or each sample size is at least 30.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

When these conditions are met, **the sampling distribution for $\overline{x}_1 - \overline{x}_2$, the difference of the sample means**, is a *normal distribution with mean and standard error* as shown.



$$\text{Mean} = \mu_{\overline{x}_1 - \overline{x}_2} = \mu_{\overline{x}_1} - \mu_{\overline{x}_2} = \mu_1 - \mu_2$$

$$\text{Standard error} = \sigma_{\overline{x}_1 - \overline{x}_2} = \sqrt{\sigma_{\overline{x}_1}^2 + \sigma_{\overline{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

TWO-SAMPLE z-TEST FOR THE DIFFERENCE BETWEEN MEANS

A **two-sample z-test** can be used to test the difference between two population means μ_1 and μ_2 when these conditions are met.

1. Both σ_1 and σ_2 are known.
2. The samples are random.
3. The samples are independent.
4. The populations are normally distributed *or* both $n_1 \geq 30$ and $n_2 \geq 30$.

The **test statistic** is $\bar{x}_1 - \bar{x}_2$. The **standardized test statistic** is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} \quad \text{where} \quad \sigma_{x_1 - x_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

Using a Two-Sample z-Test for the Difference Between Means (Independent Samples, σ_1 and σ_2 Known)

IN WORDS	IN SYMBOLS
1. Verify that σ_1 and σ_2 are known, the samples are random and independent, and either the populations are normally distributed *or* both $n_1 \geq 30$ and $n_2 \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

5. Determine the rejection region(s).

6. Find the standardized test statistic and sketch the sampling distribution.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

7. Make a decision to reject or fail to reject the null hypothesis.

If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.

8. Interpret the decision in the context of the original claim.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example A credit card watchdog (رقابة) group claims that there is a difference in the mean credit card debts of households in California and Illinois. The results of a random survey of 250 households from each state are shown in the table. The two samples are independent. Assume that $\sigma_1 = \$1045$ for California and $\sigma_2 = \$1350$ for Illinois. Do the results support the group's claim? Use $\alpha = 0.05$.

Sample Statistics for Credit Card Debt

California	Illinois
$\bar{x}_1 = \$4777$ $n_1 = 250$	$\bar{x}_2 = \$4866$ $n_2 = 250$

$\sigma_1 = \$1045$ $\sigma_2 = \$1350$

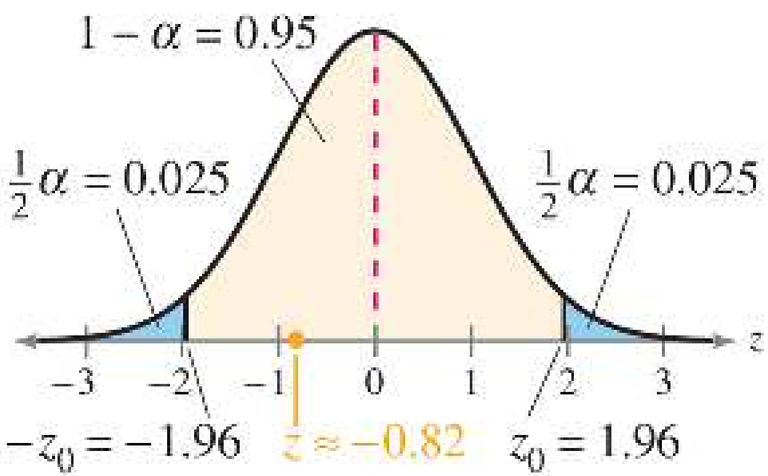
$\alpha = 0.05$

H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 \neq \mu_2$ (Claim)

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example Because the test is a two-tailed test, the critical values are $-z_0 = -1.96$ and $z_0 = 1.96$.



Sample Statistics for Credit Card Debt

California	Illinois
$\bar{x}_1 = \$4777$ $n_1 = 250$	$\bar{x}_2 = \$4866$ $n_2 = 250$

$\sigma_1 = \$1045$ $\sigma_2 = \$1350$

$\alpha = 0.05$

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$ (Claim)

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example

$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 \neq \mu_2$ (Claim)

$\sigma_1 = \$1045$
$\sigma_2 = \$1350$
$\alpha = 0.05$

Sample Statistics for Credit Card Debt

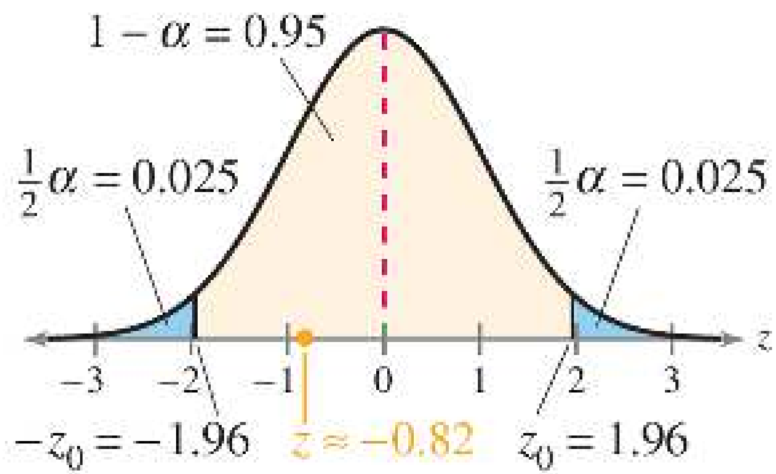
California	Illinois
$\bar{x}_1 = \$4777$ $n_1 = 250$	$\bar{x}_2 = \$4866$ $n_2 = 250$

The standardized test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{(4777 - 4866) - 0}{\sqrt{\frac{1045^2}{250} + \frac{1350^2}{250}}}$$

$$\approx -0.82.$$



TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$ (Claim)

$\sigma_1 = \$1045$

$\sigma_2 = \$1350$

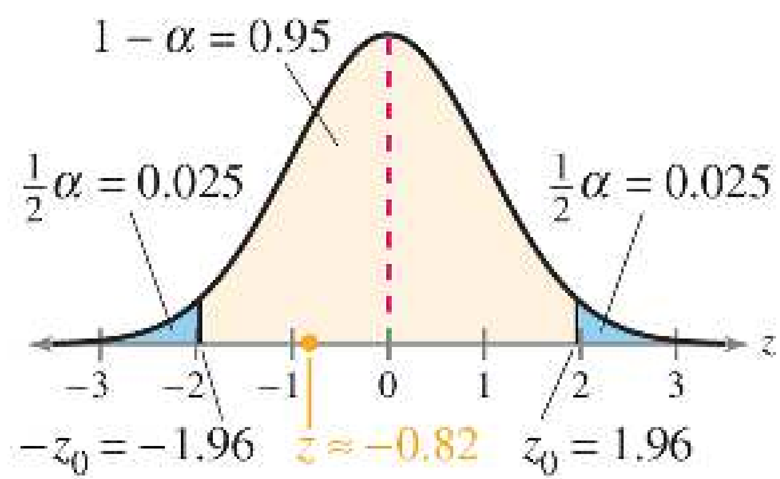
$\alpha = 0.05$

Sample Statistics for Credit Card Debt

California	Illinois
$\bar{x}_1 = \$4777$	$\bar{x}_2 = \$4866$
$n_1 = 250$	$n_2 = 250$

Because z is not in the rejection region, you fail to reject the null hypothesis.

There is not enough evidence at the 5% level of significance to support the group's claim that there is a difference in the mean credit card debts of households in California and Illinois.



TESTING A DIFFERENCE OTHER THAN ZERO

Example

Is the difference between the mean annual salaries of microbiologists in Maryland and California more than \$10,000? To decide, you select a random sample of microbiologists from each state. The results of each survey are shown in the figure. Assume the population standard deviations are $\sigma_1 = \$8795$ and $\sigma_2 = \$9250$. At $\alpha = 0.05$, what should you conclude?

Microbiologists in Maryland	Microbiologists in California
$\bar{x}_1 = \$102{,}650$	$\bar{x}_2 = \$85{,}430$
$n_1 = 42$	$n_2 = 38$
$\sigma_1 = \$8795$	$\sigma_2 = \$9250$

$$H_0 : \mu_1 - \mu_2 \leq 10{,}000$$

$$H_a : \mu_1 - \mu_2 > 10{,}000 \text{ (claim)}$$

$$\alpha = 0.05$$

TESTING A DIFFERENCE OTHER THAN ZERO

Example $H_0 : \mu_1 - \mu_2 \leq 10{,}000$

$H_a : \mu_1 - \mu_2 > 10{,}000$ (claim)

Microbiologists in Maryland	Microbiologists in California
$x_1 = \$102{,}650$	$\bar{x}_2 = \$85{,}430$
$n_1 = 42$	$n_2 = 38$
$\sigma_1 = \$8795$	$\sigma_2 = \$9250$

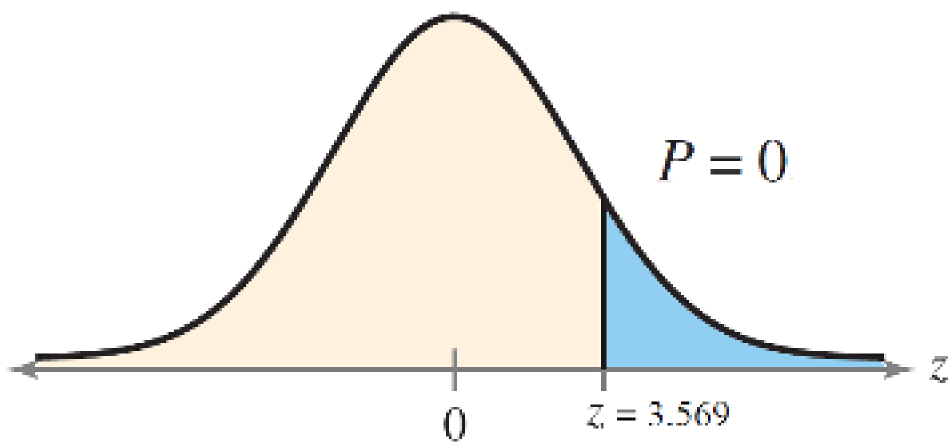
$\alpha = 0.05$

The standardized test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

$$= \frac{(102{,}650 - 85{,}430) - (10{,}000)}{\sqrt{\dfrac{(8795)^2}{42} + \dfrac{(9250)^2}{38}}}$$

$$\approx 3.569$$



TESTING A DIFFERENCE OTHER THAN ZERO

Example $H_0 : \mu_1 - \mu_2 \leq 10{,}000$

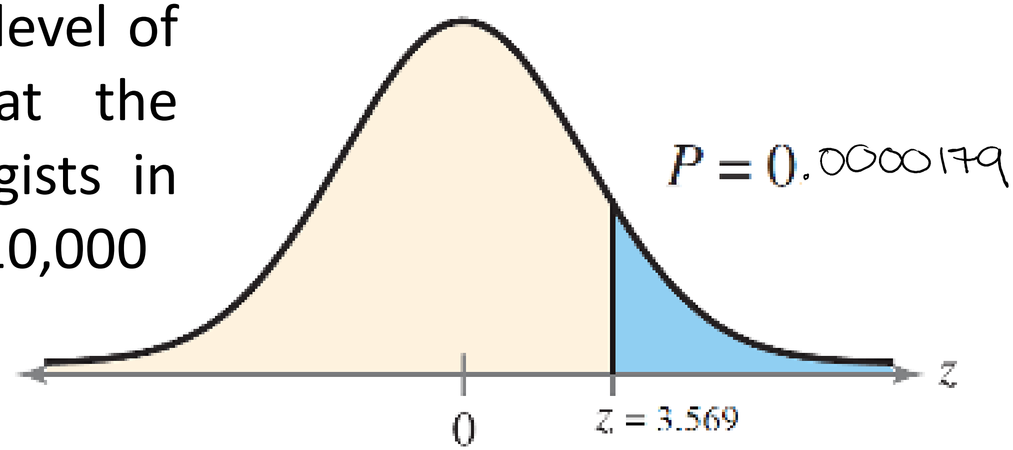
$H_a : \mu_1 - \mu_2 > 10{,}000$ (claim)

Microbiologists in Maryland	Microbiologists in California
$x_1 = \$102{,}650$	$\bar{x}_2 = \$85{,}430$
$n_1 = 42$	$n_2 = 38$
$\sigma_1 = \$8795$	$\sigma_2 = \$9250$

$\alpha = 0.05$

Because the P −value is less than $\alpha = 0.05$, you reject the null hypothesis.

There is enough evidence at the 5% level of significance to support the claim that the mean annual salaries of microbiologists in Maryland and California more than $10,000



Course: Biostatistics

Lecture No: [18]

Chapter: [8]

Hypothesis Testing with Two Sample

Section: [8.2]

Testing the Difference Between Means (Independent Samples, σ_1 and σ_2 Unknown)

THE TWO-SAMPLE t −TEST FOR THE DIFFERENCE BETWEEN MEANS

- In many real-life situations, both population standard deviations are *not* known.
- In this section, you will learn how to use a t −test to test the difference between two population means μ_1 and μ_2 using independent samples from each population when σ_1 and σ_2 are unknown.
- To use a t −test, these conditions are necessary.
  - ✓ The population standard deviations are unknown.
  - ✓ The samples are randomly selected.
  - ✓ The samples are independent.
  - ✓ The populations are normally distributed or each sample size is at least 30.

THE TWO-SAMPLE t −TEST FOR THE DIFFERENCE BETWEEN MEANS

- When these conditions are met, A **two-sample t −test** is used to test the difference between two population means μ_1 and μ_2.
- The **test statistic is** $\bar{x}_1 - \bar{x}_2$.
- The **standardized test statistic** is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

- The standard error and the degrees of freedom of the sampling distribution *depend on* whether the population variances σ_1^2 and σ_2^2 are equal.

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

1. **Variances are equal:**
   Then information from the two samples is combined to calculate a **pooled estimate** of the standard deviation $\hat{\sigma}$.

$$\hat{\sigma} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

   The *standard error* for the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is

$$s_{\bar{x}_1 - \bar{x}_2} = \hat{\sigma} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

   And d.f. $= n_1 + n_2 - 2$.

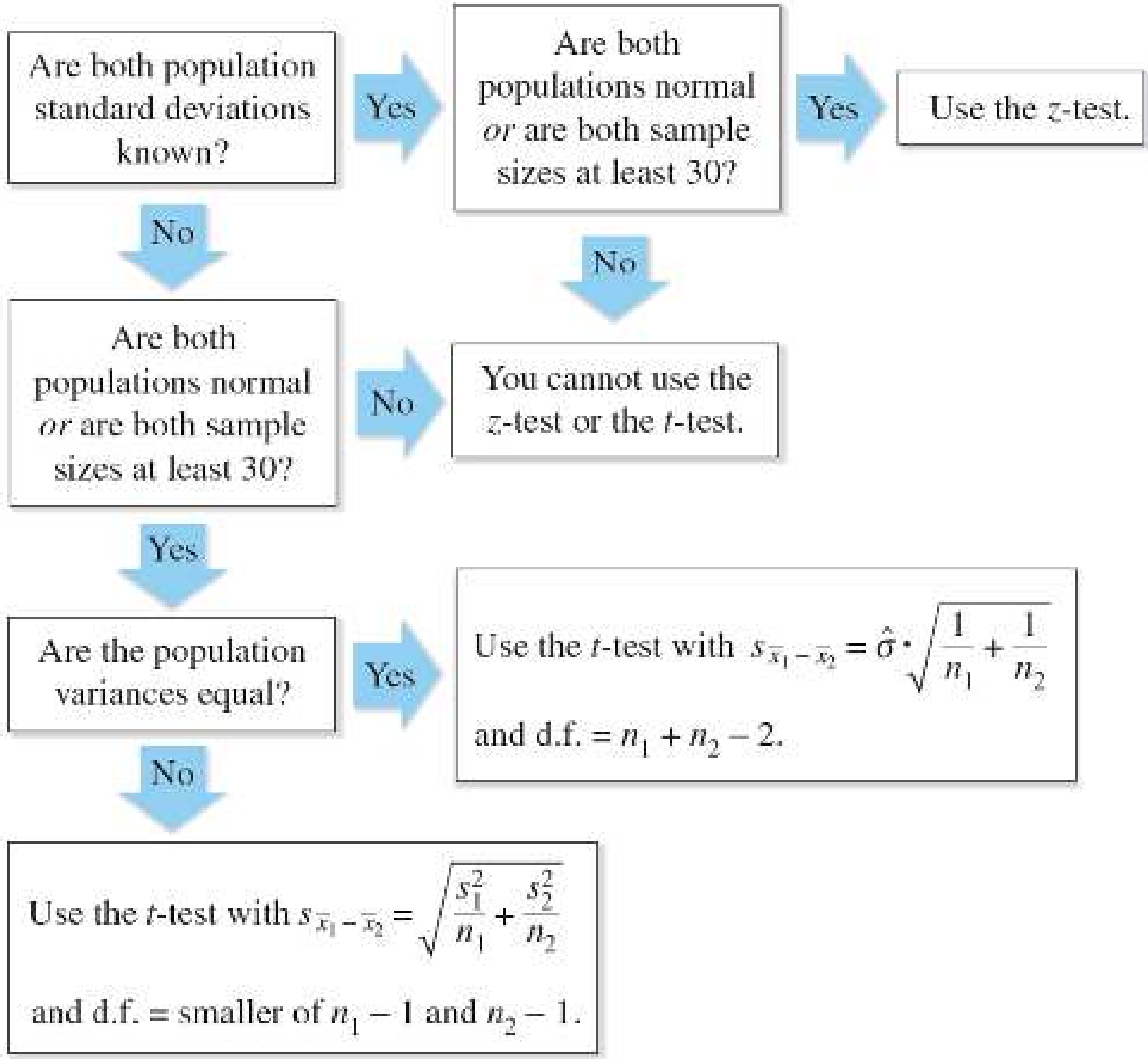
THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

2. Variances are not equal:

If the population variances are not equal, then the standard error is

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

And d.f. = smaller of $n_1 - 1$ and $n_2 - 1$.


Are both population standard deviations known?
Yes
Are both populations normal *or* are both sample sizes at least 30?
Yes
Use the z-test.
No
No
Are both populations normal *or* are both sample sizes at least 30?
No
You cannot use the z-test or the t-test.
Yes
Are the population variances equal?
Yes
Use the t-test with $s_{\bar{x}_1 - \bar{x}_2} = \hat{\sigma} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ and d.f. $= n_1 + n_2 - 2$.
No
Use the t-test with $s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ and d.f. = smaller of $n_1 - 1$ and $n_2 - 1$.

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Using a Two-Sample t-Test for the Difference Between Means (Independent Samples, σ_1 and σ_2 Unknown)

IN WORDS	IN SYMBOLS
1. Verify that σ_1 and σ_2 are unknown, the samples are random and independent, and either the populations are normally distributed *or* both $n_1 \geq 30$ and $n_2 \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the degrees of freedom.	d.f. $= n_1 + n_2 - 2$ or d.f. $=$ smaller of $n_1 - 1$ and $n_2 - 1$
5. Determine the critical value(s).	Use Table 5 in Appendix B.

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

6. Determine the rejection region(s).	
7. Find the standardized test statistic and sketch the sampling distribution.	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$
8. Make a decision to reject or fail to reject the null hypothesis.	If t is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example

The results of a state mathematics test for random samples of students taught by two different teachers at the same school are shown. Can you conclude that there is a difference in the mean mathematics test scores for the students of the two teachers? Use $\alpha = 0.10$. Assume the populations are normally distributed and the population *variances are not equal*.

Sample Statistics for State Mathematics Test Scores

Teacher 1	Teacher 2
$\bar{x}_1 = 473$	$\bar{x}_2 = 459$
$s_1 = 39.7$	$s_2 = 24.5$
$n_1 = 8$	$n_2 = 18$

$\sigma_1^2 \neq \sigma_2^2$ $\alpha = 0.10$

$$H_0: \quad \mu_1 = \mu_2$$
$$H_a: \quad \mu_1 \neq \mu_2 \quad \text{(Claim)}$$

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_1 = \mu_2$ $\sigma_1^2 \neq \sigma_2^2$

H_a: $\mu_1 \neq \mu_2$ (Claim) $\alpha = 0.10$

Because the population variances are not equal and the smaller sample size is 8, use $\text{d.f.} = 8 - 1 = 7$.

Because the test is a two-tailed test with $\text{d.f.} = 7$ and $\alpha = 0.10$, the critical values are

$$-t_0 = -1.859$$
$$t_0 = 1.859$$

Sample Statistics for State Mathematics Test Scores

Teacher 1	Teacher 2
$\bar{x}_1 = 473$	$\bar{x}_2 = 459$
$s_1 = 39.7$	$s_2 = 24.5$
$n_1 = 8$	$n_2 = 18$

	Level of confidence, c	0.80	0.90
	One tail, α	0.10	0.05
d.f.	Two tails, α	0.20	0.10
1		3.078	6.314
2		1.886	2.920
3		1.638	2.353
4		1.533	2.132
5		1.476	2.015
6		1.440	1.943
7		1.415	1.895
8		1.397	1.860

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 \neq \mu_2$ (Claim)

$\sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.10$

Sample Statistics for State Mathematics Test Scores

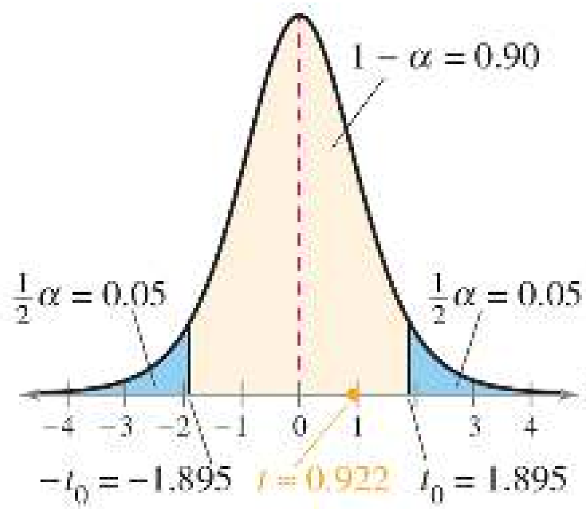
Teacher 1	Teacher 2
$\bar{x}_1 = 473$ $s_1 = 39.7$ $n_1 = 8$	$\bar{x}_2 = 459$ $s_2 = 24.5$ $n_2 = 18$

The standardized test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{(473 - 459) - 0}{\sqrt{\frac{(39.7)^2}{8} + \frac{(24.5)^2}{18}}}$$

$$\approx 0.922.$$



THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 \neq \mu_2$ (Claim)

$\sigma_1^2 \neq \sigma_2^2$

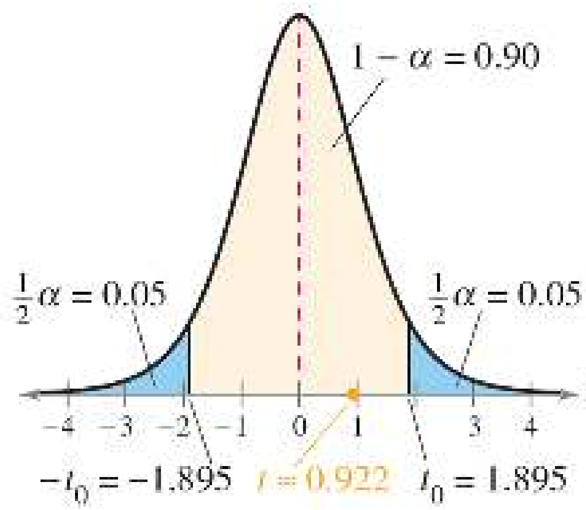
$\alpha = 0.10$

Sample Statistics for State Mathematics Test Scores

Teacher 1	Teacher 2
$\bar{x}_1 = 473$	$\bar{x}_2 = 459$
$s_1 = 39.7$	$s_2 = 24.5$
$n_1 = 8$	$n_2 = 18$

Because t is not in the rejection region, you fail to reject the null hypothesis.

There is not enough evidence at the 10% level of significance to support the claim that the mean mathematics test scores for the students of the two teachers are different.



THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example

A manufacturer claims that the mean operating cost per mile of its sedans is less than that of its leading competitor. You conduct a study using 30 randomly selected sedans from the manufacturer and 32 from the leading competitor. The results are shown. At $\alpha = 0.05$, can you support the manufacturer's claim? Assume the population **variances are equal**.

Manufacturer	Competitor
$\bar{x}_1 = \$0.52/\text{mi}$	$\bar{x}_2 = \$0.55/\text{mi}$
$s_1 = \$0.05/\text{mi}$	$s_2 = \$0.07/\text{mi}$
$n_1 = 30$	$n_2 = 32$

$\sigma_1^2 = \sigma_2^2$ $\quad$ $\alpha = 0.05$

H_0: $\mu_1 \geq \mu_2$

H_a: $\mu_1 < \mu_2$ (Claim)

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_1 \geq \mu_2$ $\sigma_1^2 = \sigma_2^2$

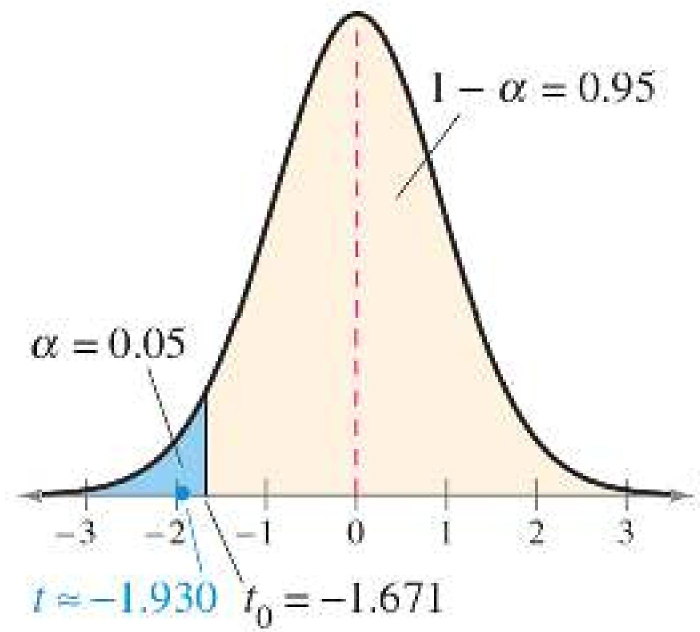
H_a: $\mu_1 < \mu_2$ (Claim) $\alpha = 0.05$

Manufacturer	Competitor
$\bar{x}_1 = \$0.52/\text{mi}$	$\bar{x}_2 = \$0.55/\text{mi}$
$s_1 = \$0.05/\text{mi}$	$s_2 = \$0.07/\text{mi}$
$n_1 = 30$	$n_2 = 32$

The population variances are equal, so

$$\text{d.f.} = n_1 + n_2 - 2 = 60$$

Because the test is a left-tailed test with $\text{d.f.} = 60$ and $\alpha = 0.05$, the critical value is $t_0 = -1.671$.



THE TWO-SAMPLE t −TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_1 \geq \mu_2$ $\sigma_1^2 = \sigma_2^2$

H_a: $\mu_1 < \mu_2$ (Claim) $\alpha = 0.05$

Manufacturer	Competitor
$\bar{x}_1 = \$0.52/\text{mi}$	$\bar{x}_2 = \$0.55/\text{mi}$
$s_1 = \$0.05/\text{mi}$	$s_2 = \$0.07/\text{mi}$
$n_1 = 30$	$n_2 = 32$

The standard error

$$s_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= \sqrt{\frac{(30-1)(0.05)^2 + (32-1)(0.07)^2}{30+32-2}} \cdot \sqrt{\frac{1}{30} + \frac{1}{32}}$$

$$\approx 0.0155416$$

THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_1 \geq \mu_2$ $\sigma_1^2 = \sigma_2^2$

H_a: $\mu_1 < \mu_2$ (Claim) $\alpha = 0.05$

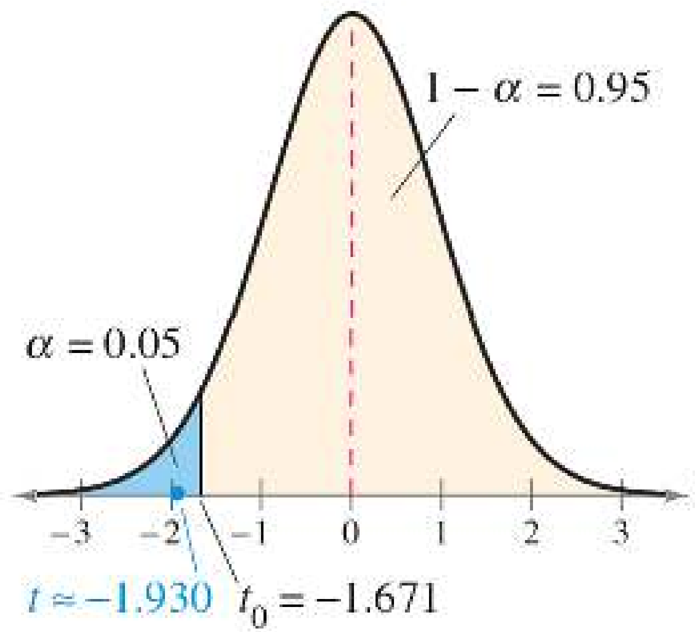
Manufacturer	Competitor
$\bar{x}_1 = \$0.52/\text{mi}$	$\bar{x}_2 = \$0.55/\text{mi}$
$s_1 = \$0.05/\text{mi}$	$s_2 = \$0.07/\text{mi}$
$n_1 = 30$	$n_2 = 32$

The standardized test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

$$\approx \frac{(0.52 - 0.55) - 0}{0.0155416}$$

$$\approx -1.930.$$



THE TWO-SAMPLE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

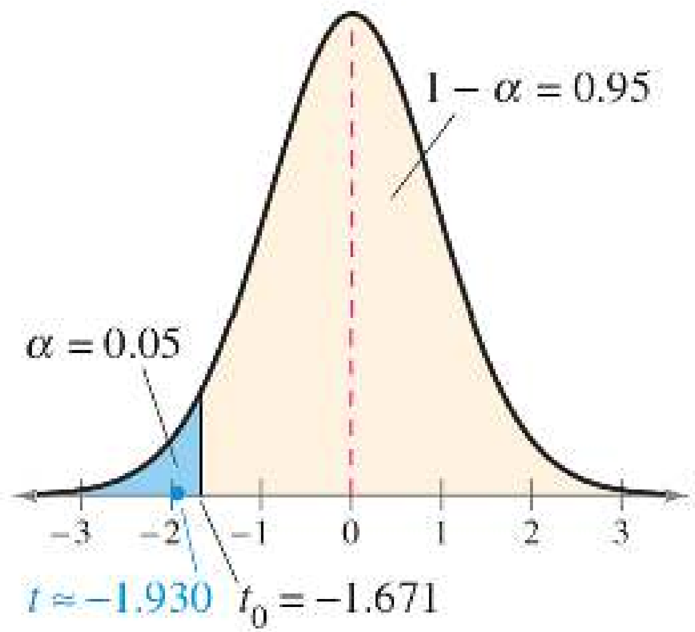
Example H_0: $\mu_1 \geq \mu_2$ $\sigma_1^2 = \sigma_2^2$

H_a: $\mu_1 < \mu_2$ (Claim) $\alpha = 0.05$

Manufacturer	Competitor
$\bar{x}_1 = \$0.52/\text{mi}$	$\bar{x}_2 = \$0.55/\text{mi}$
$s_1 = \$0.05/\text{mi}$	$s_2 = \$0.07/\text{mi}$
$n_1 = 30$	$n_2 = 32$

Because t is in the rejection region, you reject the null hypothesis.

There is enough evidence at the 5% level of significance to support the manufacturer's claim that the mean operating cost per mile of its sedans is less than that of its competitor's.



Course: Biostatistics Lecture No: [18]

Chapter: [8]

Hypothesis Testing with Two Sample

Section: [8.3]

Testing the Difference Between Means (dependent Samples)

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

- To perform a two-sample hypothesis test with dependent samples, you will use a different technique.
- You will first find the difference d for each data pair.

$$d = \begin{bmatrix}\text{data entry in} \\ \text{first sample}\end{bmatrix} - \begin{bmatrix}\text{corresponding data} \\ \text{entry in second sample}\end{bmatrix}$$

- The **test statistic** is the mean $\bar{d}$ of these differences.

$$\bar{d} = \sum \frac{d}{n}$$

THE t −TEST FOR THE DIFFERENCE BETWEEN MEANS

- These conditions are *necessary* to conduct the test.
  - ✓ The samples are randomly selected.
  - ✓ The samples are dependent (paired).
  - ✓ The populations are normally distributed or the number n of pairs of data is at least 30.
- When these conditions are met, **the sampling distribution for $\bar{d}$**, the mean of the differences of the paired data entries in the dependent samples, is approximated by a t −distribution with $n - 1$ degrees of freedom, where n is the number of data pairs.

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Symbol	Description
n	The number of pairs of data
d	The difference between entries in a data pair
μ_d	The hypothesized mean of the differences of paired data in the population
$\bar{d}$	The mean of the differences between the paired data entries in the dependent samples $\bar{d} = \frac{\Sigma d}{n}$
s_d	The standard deviation of the differences between the paired data entries in the dependent samples $s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n - 1}}$ $s_d = \sqrt{\frac{\Sigma d^2 - \left[\frac{(\Sigma d)^2}{n}\right]}{n - 1}}$.

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

A t-test can be used to test the difference of two population means when these conditions are met.

1. The samples are random.
2. The samples are dependent (paired).
3. The populations are normally distributed *or* $n \geq 30$.

The **test statistic** is $\bar{d} = \dfrac{\Sigma d}{n}$

and the **standardized test statistic** is $t = \dfrac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$.

The degrees of freedom are d.f. $= n - 1$.

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Using the t-Test for the Difference Between Means (Dependent Samples)

IN WORDS	IN SYMBOLS
1. Verify that the samples are random and dependent, and either the populations are normally distributed *or* $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= n - 1$
5. Determine the critical value(s).	Use Table 5 in Appendix B.

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

6. Determine the rejection region(s).

7. Calculate $\bar{d}$ and s_d.

$$\bar{d} = \frac{\Sigma d}{n}$$

$$s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n - 1}}$$

8. Find the standardized test statistic and sketch the sampling distribution.

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$$

9. Make a decision to reject or fail to reject the null hypothesis.

If t is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.

10. Interpret the decision in the context of the original claim.

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example

A shoe manufacturer claims that athletes can increase their vertical jump heights using the manufacturer's training shoes. *The vertical jump heights of eight randomly selected athletes are measured. After the athletes have used the shoes for 8 months, their vertical jump heights are measured again*. The vertical jump heights (in inches) for each athlete are shown in the table. At $\alpha = 0.10$, is there enough evidence to support the manufacturer's claim? Assume the vertical jump heights are normally distributed.

Athlete	1	2	3	4	5	6	7	8
Vertical jump height (before using shoes)	24	22	25	28	35	32	30	27
Vertical jump height (after using shoes)	26	25	25	29	33	34	35	30

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example $\alpha = 0.10$

The manufacturer claims that an athlete's vertical jump height before using the shoes will be less than the athlete's vertical jump height after using the shoes.

Before	After
24	26
22	25
25	25
28	29
35	33
32	34
30	35
27	30

Each difference is given by $d = \begin{bmatrix}\text{jump height}\\\text{before shoes}\end{bmatrix} - \begin{bmatrix}\text{jump height}\\\text{after shoes}\end{bmatrix}$

The null and alternative hypotheses are

$$H_0: \quad \mu_d \geq 0$$
$$H_a: \quad \mu_d < 0 \quad (\text{Claim})$$

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example $\alpha = 0.10$

Before	After
24	26
22	25
25	25
28	29
35	33
32	34
30	35
27	30

$$d = \begin{bmatrix} \text{jump height} \\ \text{before shoes} \end{bmatrix} - \begin{bmatrix} \text{jump height} \\ \text{after shoes} \end{bmatrix}$$

H_0: $\mu_d \geq 0$

H_a: $\mu_d < 0$ (Claim)

Because the test is a left-tailed test, $\alpha = 0.10$, and $\text{d.f.} = 8 - 1 = 7$, the critical value is $t_0 = -1.415$.

	Level of confidence, c	0.80	0.90
	One tail, α	0.10	0.05
d.f.	Two tails, α	0.20	0.10
1		3.078	6.314
2		1.886	2.920
3		1.638	2.353
4		1.533	2.132
5		1.476	2.015
6		1.440	1.943
7		1.415	1.895
8		1.397	1.860

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example $\alpha = 0.10$

H_0: $\mu_d \geq 0$

H_a: $\mu_d < 0$ (Claim)

Before	After	d	d^2
24	26	−2	4
22	25	−3	9
25	25	0	0
28	29	−1	1
35	33	2	4
32	34	−2	4
30	35	−5	25
27	30	−3	9
		$\Sigma = -14$	$\Sigma = 56$

$$d = \begin{bmatrix}\text{jump height} \\ \text{before shoes}\end{bmatrix} - \begin{bmatrix}\text{jump height} \\ \text{after shoes}\end{bmatrix}$$

$$\bar{d} = \sum \frac{d}{n} = \frac{-14}{8} = -1.75$$

$$s_d = \sqrt{\frac{\Sigma(d^2) - \left[\frac{(\Sigma d)^2}{n}\right]}{n-1}} = \sqrt{\frac{56 - \left[\frac{(-14)^2}{8}\right]}{7}}$$

$$\approx 2.1213$$

Because the test is a left-tailed test, $\alpha = 0.10$, and d.f. $= 8 - 1 = 7$, the critical value is $t_0 = -1.415$.

THE t −TEST FOR THE DIFFERENCE BETWEEN MEANS

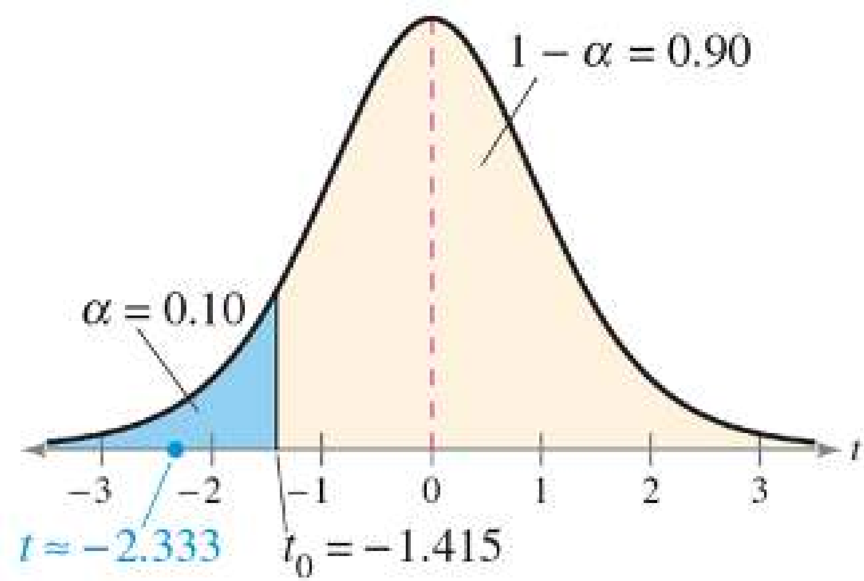
Example H_0: $\mu_d \geq 0$ $\alpha = 0.10$ $\bar{d} = -1.75$ $s_d \approx 2.1213$

H_a: $\mu_d < 0$ (Claim)

Because the test is a left-tailed test, $\alpha = 0.10$, and d.f. $= 8 - 1 = 7$, the critical value is $t_0 = -1.415$.

The standardized test statistic is

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} \approx \frac{-1.75 - 0}{2.1213 / \sqrt{8}} \approx -2.333.$$



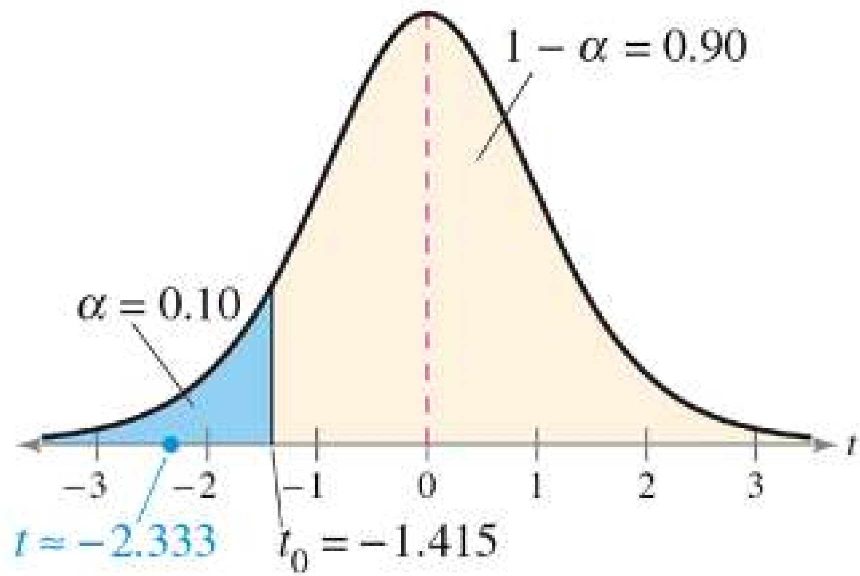
THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_d \geq 0$ $\quad \alpha = 0.10 \quad \bar{d} = -1.75 \quad s_d \approx 2.1213$

H_a: $\mu_d < 0$ (Claim)

Because the test is a left-tailed test, $\alpha = 0.10$, and d.f. $= 8 - 1 = 7$, the critical value is $t_0 = -1.415$.

Because t is in the rejection region, you reject the null hypothesis.



There is enough evidence at the 10% level of significance to support the shoe manufacturer's claim that athletes can increase their vertical jump heights using the manufacturer's training shoes.

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example The campaign staff (موظفو حملة) for a state legislator (مشرع دولة) wants to determine whether the legislator's performance rating (0–100) has changed from last year to this year. The table below shows the legislator's performance ratings from the same 16 randomly selected voters for last year and this year. At $\alpha = 0.01$, is there enough evidence to conclude that the legislator's performance rating has changed? Assume the performance ratings are normally distributed.

Voter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Rating (last year)	60	54	78	84	91	25	50	65	68	81	75	45	62	79	58	63
Rating (this year)	56	48	70	60	85	40	40	55	80	75	78	50	50	85	53	60

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example $n = 16$ $\alpha = 0.01$

If there is a change in the legislator's rating, then there will be a difference between last year's ratings and this year's ratings.

Because the legislator wants to determine whether there is a difference, the null and alternative hypotheses are

$$H_0: \quad \mu_d = 0$$
$$H_a: \quad \mu_d \neq 0 \quad \text{(Claim)}$$

Before	After	d	d^2
60	56	4	16
54	48	6	36
78	70	8	64
84	60	24	576
91	85	6	36
25	40	−15	225
50	40	10	100
65	55	10	100
68	80	−12	144
81	75	6	36
75	78	−3	9
45	50	−5	25
62	50	12	144
79	85	−6	36
58	53	5	25
63	60	3	9
		$\Sigma = 53$	$\Sigma = 1581$

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_d = 0$ $\alpha = 0.01$

H_a: $\mu_d \neq 0$ (Claim) $n = 16$

Before	After	d	d^2
60	56	4	16
54	48	6	36
78	70	8	64
84	60	24	576
91	85	6	36
25	40	−15	225
50	40	10	100
65	55	10	100
68	80	−12	144
81	75	6	36
75	78	−3	9
45	50	−5	25
62	50	12	144
79	85	−6	36
58	53	5	25
63	60	3	9
		$\Sigma = 53$	$\Sigma = 1581$

Because the test is a two-tailed test with

$$\alpha = 0.01$$

$$\text{d.f.} = 16 - 1 = 15$$

the critical values are

$$-t_0 = -2.947$$

$$t_0 = 2.947$$

	Level of confidence, c	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_d = 0$

H_a: $\mu_d \neq 0$ (Claim)

$\alpha = 0.01$

$n = 16$

$-t_0 = -2.947$

$t_0 = 2.947$

$$d = \begin{bmatrix}\text{Rating Last} \\ \text{Year}\end{bmatrix} - \begin{bmatrix}\text{Rating This} \\ \text{Year}\end{bmatrix}$$

$$\bar{d} = \sum \frac{d}{n} = \frac{53}{16} = 3.3125$$

$$s_d = \sqrt{\frac{\sum(d^2) - \left[\frac{(\sum d)^2}{n}\right]}{n-1}} = \sqrt{\frac{1581 - \left[\frac{(53)^2}{16}\right]}{15}}$$

≈ 9.6797

Before	After	d	d^2
60	56	4	16
54	48	6	36
78	70	8	64
84	60	24	576
91	85	6	36
25	40	−15	225
50	40	10	100
65	55	10	100
68	80	−12	144
81	75	6	36
75	78	−3	9
45	50	−5	25
62	50	12	144
79	85	−6	36
58	53	5	25
63	60	3	9
		$\Sigma = 53$	$\Sigma = 1581$

THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$ (Claim)

$\alpha = 0.01$
$n = 16$
$-t_0 = -2.947$
$t_0 = 2.947$

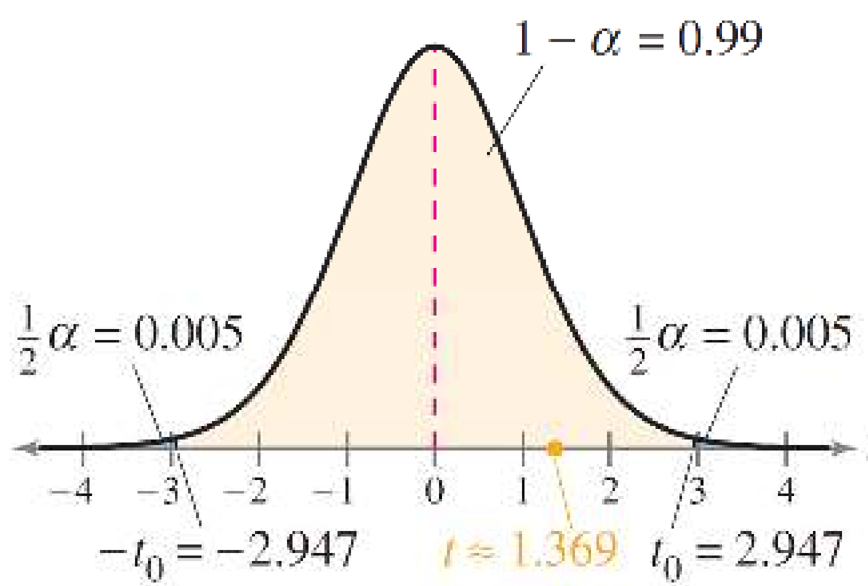
$\bar{d} = 3.3125$ $s_d \approx 9.6797$

The standardized test statistic is

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$$

$$\approx \frac{3.3125 - 0}{9.6797 / \sqrt{16}}$$

$$\approx 1.369.$$



THE t –TEST FOR THE DIFFERENCE BETWEEN MEANS

Example H_0: $\mu_d = 0$

H_a: $\mu_d \neq 0$ (Claim)

$\alpha = 0.01$

$n = 16$

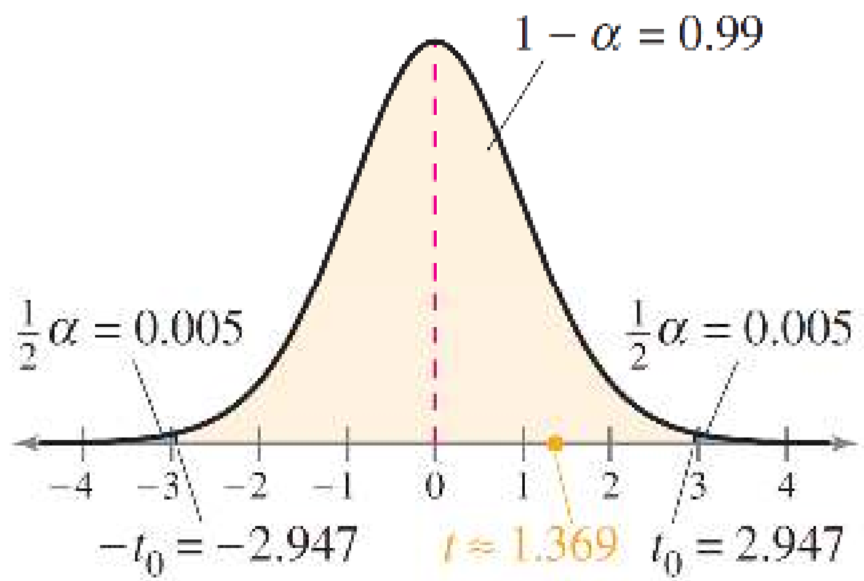
$-t_0 = -2.947$

$t_0 = 2.947$

$\bar{d} = 3.3125$ $s_d \approx 9.6797$

Because t is not in the rejection region, you fail to reject the null hypothesis.

There is not enough evidence at the 1% level of significance to conclude that the legislator's performance rating has changed.



Course: Biostatistics

Lecture No: [19]

Chapter: [8]

Hypothesis Testing with Two Sample

Section: [8.4]

Testing the Difference Between Proportions

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- In this section, you will learn how to use a z –test to test the difference between two population proportions p_1 and p_2 using a sample proportion from each population.
- If a claim is about two population parameters p_1 and p_2, then some possible pairs of null and alternative hypotheses are

$$H_0: p_1 = p_2 \qquad H_0: p_1 \leq p_2 \qquad H_0: p_1 \geq p_2$$
$$H_a: p_1 \neq p_2 \qquad H_a: p_1 > p_2 \qquad H_a: p_1 < p_2$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- These conditions are necessary to use a z –test to test such a difference.
  - ✓ The samples are randomly selected.
  - ✓ The samples are independent.
  - ✓ The samples are large enough to use a normal sampling distribution. That is, $n_1 p_1 \geq 5$, $n_1 q_1 \geq 5$, $n_2 p_2 \geq 5$ and $n_2 q_2 \geq 5$.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- When these conditions are met, the sampling distribution for $\hat{p}_1 - \hat{p}_2$, the difference between the sample proportions, is a normal distribution with mean

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$$

and standard error

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- Notice that you need to know the population proportions to calculate the standard error.
- You can calculate a weighted estimate of p_1 and p_2 using

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

- With the weighted estimate $\bar{p}$, the standard error of the sampling distribution for $\hat{p}_1 + \hat{p}_2$is

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \text{ where } \bar{q} = 1 - \bar{p}$$

TWO-SAMPLE z −TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- Also, you need to know the population proportions to verify that the samples are large enough to be approximated by the normal distribution.
- But when determining whether the z −test can be used for the difference between proportions for a binomial experiment, you should use $\bar{p}$ in place of p_1 and p_2 and use $\bar{q}$ in place of q_1 and q_2.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

A two-sample z-test is used to test the difference between two population proportions p_1 and p_2 when these conditions are met.

1. The samples are random.
2. The samples are independent.
3. The quantities $n_1\bar{p}$, $n_1\bar{q}$, $n_2\bar{p}$, and $n_2\bar{q}$ are at least 5.

The **test statistic** is $\hat{p}_1 - \hat{p}_2$. The **standardized test statistic** is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$ and $\bar{q} = 1 - \bar{p}$.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Using a Two-Sample z-Test for the Difference Between Proportions

IN WORDS	IN SYMBOLS
1. Verify that the samples are random and independent.	
2. Find the weighted estimate of p_1 and p_2. Verify that $n_1\bar{p}$, $n_1\bar{q}$, $n_2\bar{p}$, and $n_2\bar{q}$ are at least 5.	$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}, \bar{q} = 1 - \bar{p}$
3. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
4. Specify the level of significance.	Identify α.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

5. Determine the critical value(s). — Use Table 4 in Appendix B.
6. Determine the rejection region(s).
7. Find the standardized test statistic and sketch the sampling distribution. — $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$
8. Make a decision to reject or fail to reject the null hypothesis. — If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.

TWO-SAMPLE z −TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example A medical research team conducted a study to test the effect of a cholesterol reducing medication. At the end of the study, the researchers found that of the 4700 randomly selected subjects who took the medication, 301 died of heart disease. Of the 4300 randomly selected subjects who took a placebo, 357 died of heart disease. At $\alpha = 0.01$, can you support the claim that the death rate due to heart disease is lower for those who took the medication than for those who took the placebo?

$n_1 = 4700$ $n_2 = 4300$ $\alpha = 0.01$

$x_1 = 301$ $x_2 = 357$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example The death rate due to heart disease is lower for those who took the medication than for those who took the placebo.

$n_1 = 4700 \quad x_1 = 301$

$n_2 = 4300 \quad x_2 = 357$

$\alpha = 0.01$

The samples are random and independent.

The weighted estimate of p_1 and p_2 is $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{658}{9000} \approx 0.0731$

$$\bar{q} = 1 - \bar{p} = 0.9269$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example The death rate due to heart disease is lower for those who took the medication than for those who took the placebo.

$n_1 = 4700 \quad x_1 = 301$

$n_2 = 4300 \quad x_2 = 357$

$\bar{p} = 0.0731 \quad \alpha = 0.01$

$\bar{q} = 0.9269$

Because

$$n_1\bar{p} = (4700)(0.0731) \geq 5$$

$$n_1\bar{q} = (4700)(0.9269) \geq 5$$

$$n_2\bar{p} = (4300)(0.0731) \geq 5$$

$$n_2\bar{q} = (4300)(0.9269) \geq 5$$

you can use a two-sample z –test.

H_0: $p_1 \geq p_2$

H_a: $p_1 < p_2$ (Claim)

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{301}{4700} = 0.0640$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{357}{4300} = 0.0830$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example H_0: $p_1 \geq p_2$

H_a: $p_1 < p_2$ (Claim)

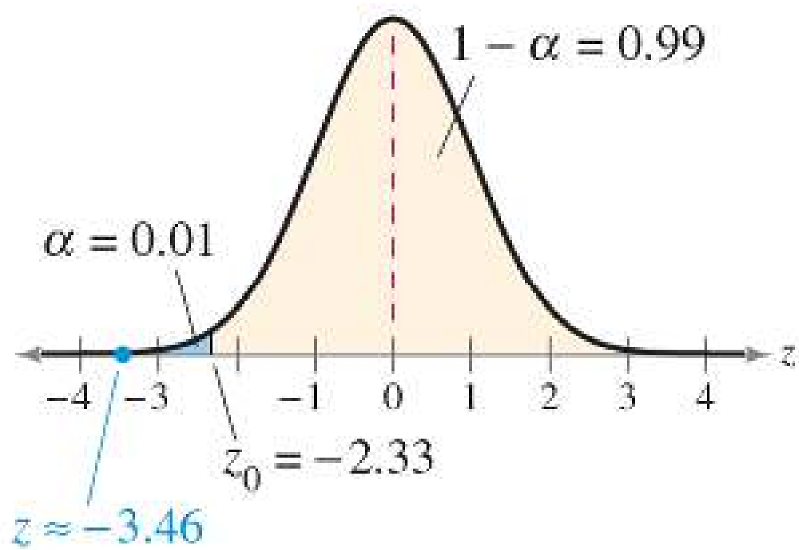
$n_1 = 4700$ $\bar{p} = 0.0731$

$n_2 = 4300$ $\bar{q} = 0.9269$

$\alpha = 0.01$ $\hat{p}_1 = 0.0640$

$\hat{p}_2 = 0.0830$

Because the test is left-tailed and the level of significance is $\alpha = 0.01$, the critical value is $z_0 = -2.33$.



TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example H_0: $p_1 \geq p_2$
H_a: $p_1 < p_2$ (Claim)

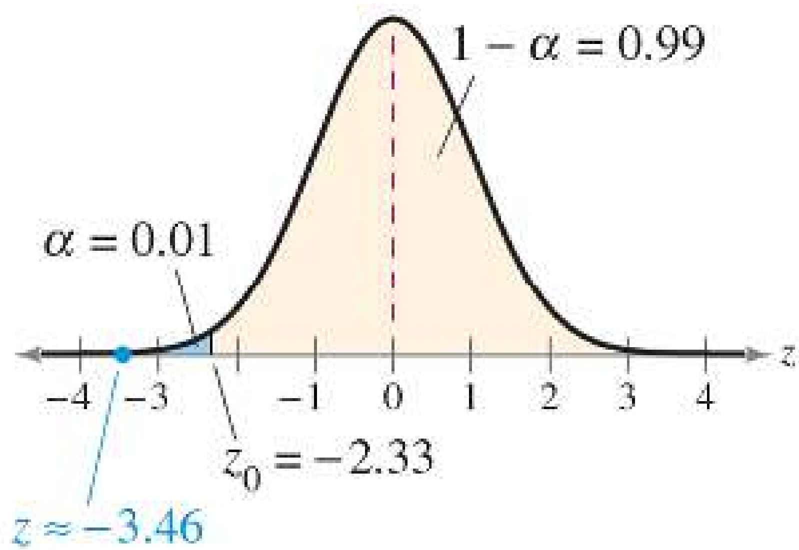
$n_1 = 4700$ $\bar{p} = 0.0731$
$n_2 = 4300$ $\bar{q} = 0.9269$
$\alpha = 0.01$ $\hat{p}_1 = 0.0640$
$\hat{p}_2 = 0.0830$

The standardized test statistic is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\approx \frac{(0.0640 - 0.0830) - 0}{\sqrt{(0.0731)(0.9269)\left(\frac{1}{4700} + \frac{1}{4300}\right)}}$$

$$\approx -3.46$$



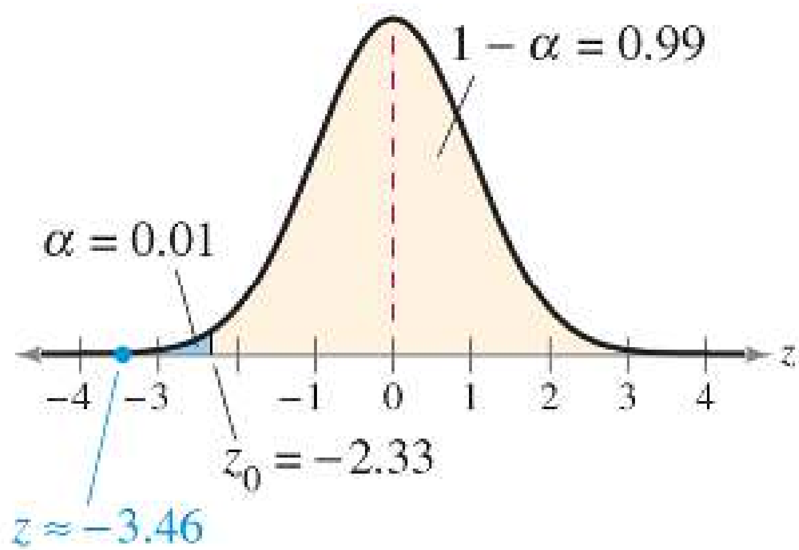
TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example H_0: $p_1 \geq p_2$
H_a: $p_1 < p_2$ (Claim)

$n_1 = 4700$ $\bar{p} = 0.0731$
$n_2 = 4300$ $\bar{q} = 0.9269$
$\alpha = 0.01$ $\hat{p}_1 = 0.0640$
$\hat{p}_2 = 0.0830$

Because z is in the rejection region, you reject the null hypothesis.

There is enough evidence at the 1% level of significance to support the claim that the death rate due to heart disease is lower for those who took the medication than for those who took the placebo.



Course: Biostatistics Lecture No: [20]

Chapter: [8]

Hypothesis Testing with Two Sample

Section: [8.4]

Testing the Difference Between Proportions

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- In this section, you will learn how to use a z –test to test the difference between two population proportions p_1 and p_2 using a sample proportion from each population.
- If a claim is about two population parameters p_1 and p_2, then some possible pairs of null and alternative hypotheses are

$$\begin{array}{lll} H_0: p_1 = p_2 & H_0: p_1 \leq p_2 & H_0: p_1 \geq p_2 \\ H_a: p_1 \neq p_2 & H_a: p_1 > p_2 & H_a: p_1 < p_2 \end{array}$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- These conditions are necessary to use a z –test to test such a difference.
  - ✓ The samples are randomly selected.
  - ✓ The samples are independent.
  - ✓ The samples are large enough to use a normal sampling distribution. That is, $n_1p_1 \geq 5$, $n_1q_1 \geq 5$, $n_2p_2 \geq 5$ and $n_2q_2 \geq 5$.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- When these conditions are met, the sampling distribution for $\hat{p}_1 - \hat{p}_2$, the difference between the sample proportions, is a normal distribution with mean

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$$

and standard error

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- Notice that you need to know the population proportions to calculate the standard error.
- You can calculate a weighted estimate of p_1 and p_2 using

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

- With the weighted estimate $\bar{p}$, the standard error of the sampling distribution for $\hat{p}_1 - \hat{p}_2$ is

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \text{ where } \bar{q} = 1 - \bar{p}$$

TWO-SAMPLE z −TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

- Also, you need to know the population proportions to verify that the samples are large enough to be approximated by the normal distribution.
- But when determining whether the z −test can be used for the difference between proportions for a binomial experiment, you should use $\bar{p}$ in place of p_1 and p_2 and use $\bar{q}$ in place of q_1 and q_2.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

A two-sample z-test is used to test the difference between two population proportions p_1 and p_2 when these conditions are met.

1. The samples are random.
2. The samples are independent.
3. The quantities $n_1\overline{p}$, $n_1\overline{q}$, $n_2\overline{p}$, and $n_2\overline{q}$ are at least 5.

The **test statistic** is $\hat{p}_1 - \hat{p}_2$. The **standardized test statistic** is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\overline{p}\,\overline{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\overline{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$ and $\overline{q} = 1 - \overline{p}$.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Using a Two-Sample z-Test for the Difference Between Proportions

IN WORDS	IN SYMBOLS
1. Verify that the samples are random and independent.	
2. Find the weighted estimate of p_1 and p_2. Verify that $n_1\overline{p}$, $n_1\overline{q}$, $n_2\overline{p}$, and $n_2\overline{q}$ are at least 5.	$\overline{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$, $\overline{q} = 1 - \overline{p}$
3. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
4. Specify the level of significance.	Identify α.

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

5. Determine the critical value(s).	Use Table 4 in Appendix B.
6. Determine the rejection region(s).	
7. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$
8. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

TWO-SAMPLE z −TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example A medical research team conducted a study to test the effect of a cholesterol reducing medication. At the end of the study, the researchers found that of the 4700 randomly selected subjects who took the medication, 301 died of heart disease. Of the 4300 randomly selected subjects who took a placebo, 357 died of heart disease. At $\alpha = 0.01$, can you support the claim that the death rate due to heart disease is lower for those who took the medication than for those who took the placebo?

$n_1 = 4700$ $n_2 = 4300$ $\alpha = 0.01$

$x_1 = 301$ $x_2 = 357$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example The death rate due to heart disease is lower for those who took the medication than for those who took the placebo.

$n_1 = 4700 \quad x_1 = 301$

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$\alpha = 0.01$

The samples are random and independent.

The weighted estimate of p_1 and p_2 is $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{658}{9000} \approx 0.0731$

$$\bar{q} = 1 - \bar{p} = 0.9269$$

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example The death rate due to heart disease is lower for those who took the medication than for those who took the placebo.

$n_1 = 4700 \quad x_1 = 301$

$n_2 = 4300 \quad x_2 = 357$

$\bar{p} = 0.0731 \quad \alpha = 0.01$

$\bar{q} = 0.9269$

Because $n_1\bar{p} = (4700)(0.0731) \geq 5$

$n_1\bar{q} = (4700)(0.9269) \geq 5$

$n_2\bar{p} = (4300)(0.0731) \geq 5$

$n_2\bar{q} = (4300)(0.9269) \geq 5$

you can use a two-sample z –test.

$H_0: \quad p_1 \geq p_2$

$H_a: \quad p_1 < p_2 \quad$ (Claim)

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{301}{4700} = 0.0640$$

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TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example H_0: $p_1 \geq p_2$

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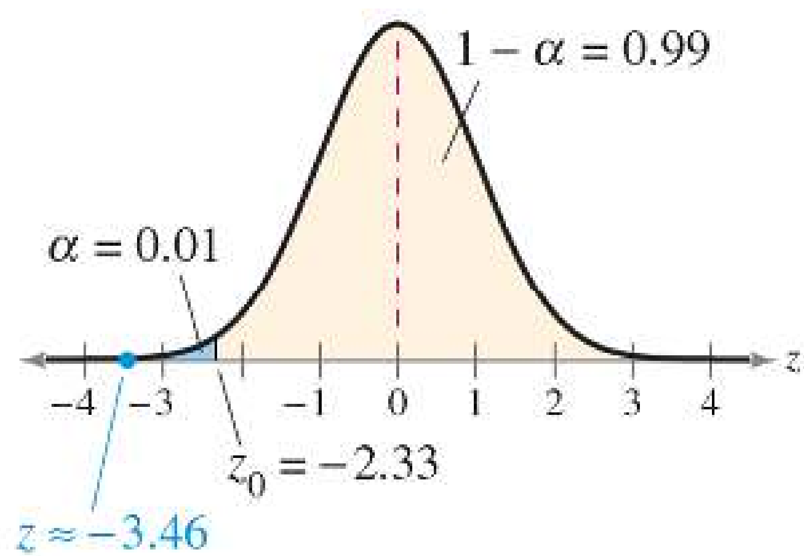

TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

Example H_0: $p_1 \geq p_2$
H_a: $p_1 < p_2$ (Claim)

$n_1 = 4700$ $\bar{p} = 0.0731$
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$\alpha = 0.01$ $\hat{p}_1 = 0.0640$
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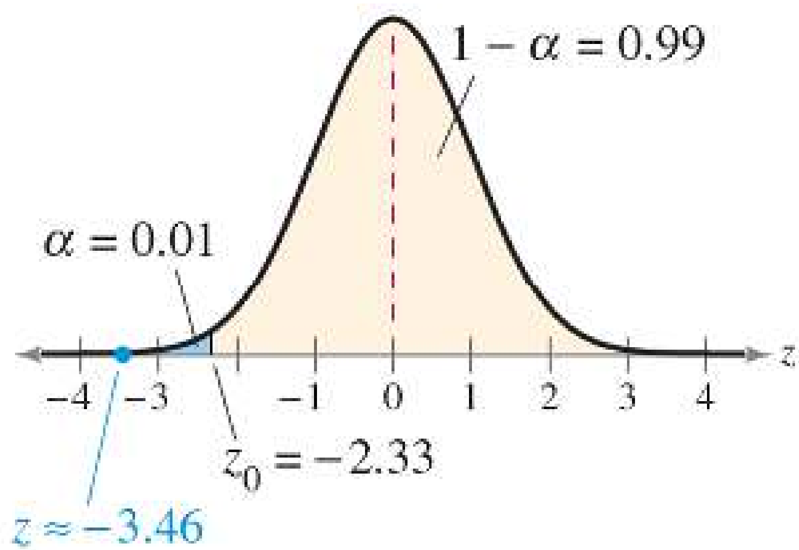
The standardized test statistic is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\approx \frac{(0.0640 - 0.0830) - 0}{\sqrt{(0.0731)(0.9269)\left(\frac{1}{4700} + \frac{1}{4300}\right)}}$$

$$\approx -3.46$$



TWO-SAMPLE z –TEST FOR THE DIFFERENCE BETWEEN PROPORTIONS

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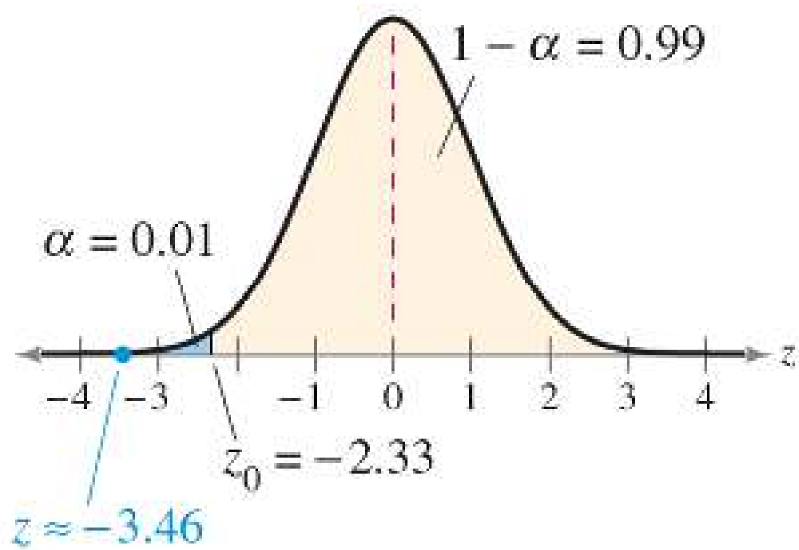
Because z is in the rejection region, you reject the null hypothesis.

There is enough evidence at the 1% level of significance to support the claim that the death rate due to heart disease is lower for those who took the medication than for those who took the placebo.



Course: Biostatistics Lecture No: [21]

Chapter: [10]

Chi-Square Tests and the F –Distribution

Section: [10.1]

Goodness-of-Fit Test

WHERE YOU'RE GOING

- In Chapter 8, you learned how to test a hypothesis that compares two populations by basing your decisions on sample statistics and their distributions.
- In this chapter, you will learn *how to test a hypothesis that compares three or more populations*.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- **WHAT YOU SHOULD LEARN?**

  How to use the chi-square distribution to test whether a frequency distribution fits an expected distribution.

- **Example:**
  - ✓ A tax preparation company wants to determine the proportions of people who used different methods to prepare their taxes.
  - ✓ Accountant, By hand, Computer software, Friend/family, Tax preparation service.
  - ✓ To determine these proportions, the company can perform a *multinomial experiment*.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- A **multinomial experiment** is a probability experiment consisting of a fixed number of independent trials in which there are more than two possible outcomes for each trial.
- The probability of each outcome is fixed, and each outcome is classified into categories.

Distribution of tax preparation methods	
Accountant	24%
By hand	20%
Computer software	35%
Friend/family	6%
Tax preparation service	15%

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- To *compare the distributions*, you can perform a chi-square goodness-of-fit test.
- **Definition:** A chi-square goodness-of-fit test is used to test whether a frequency distribution **fits** an expected distribution.
- To begin a goodness-of-fit test, you must first state a null and an alternative hypothesis.
- Generally, the *null hypothesis* states that *the frequency distribution fits an expected distribution* and the *alternative hypothesis* states that *the frequency distribution does not fit the expected distribution*.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- Back to our tax preparation company example:

  H_0: The expected distribution of tax preparation methods is 24% by accountant, 20% by hand, 35% by computer software, 6% by friend or family, and 15% by tax preparation service. (**Claim**)

  H_a: The distribution of tax preparation methods differs from the expected distribution.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- To calculate the *test statistic* for the chi-square goodness-of-fit test, you can use **observed frequencies** and **expected frequencies**.
- To calculate the expected frequencies, you must assume the null hypothesis is true.
- **Definition:**

  The **observed frequency O** of a category is the frequency for the category observed in the sample data.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- **Definition:**

  The **expected frequency E** of a category is the calculated frequency for the category.

  ✓ Expected frequencies are found by using the expected (or hypothesized) distribution and the sample size.

  ✓ The expected frequency for the i^{th} category is $E_i = np_i$ where n is the number of trials (the sample size) and p_i is the assumed probability of the i^{th} category.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

A tax preparation company randomly selects 300 adults and asks them *how they prepare their taxes*. The results are shown at the right. Find the observed frequency and the expected frequency (using the distribution on the bottom corner) for each tax preparation method.

Observed

Survey results (n = 300)	
Accountant	61
By hand	42
Computer software	112
Friend/family	29
Tax preparation service	56

Distribution of tax preparation methods	
Accountant	24%
By hand	20%
Computer software	35%
Friend/family	6%
Tax preparation service	15%

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

Tax preparation method	p	O	E
Accountant	0.24	61	(300)(0.24)=72
By hand	0.20	42	(300)(0.20)=60
Computer Software	0.35	112	(300)(0.35)=105
Friend Family	0.06	29	(300)(0.06)=18
Tax preparation service	0.15	56	(300)(0.15)=45

Observed

Survey results (n = 300)	
Accountant	61
By hand	42
Computer software	112
Friend/family	29
Tax preparation service	56

Distribution of tax preparation methods	
Accountant	24%
By hand	20%
Computer software	35%
Friend/family	6%
Tax preparation service	15%

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- To perform a chi-square goodness-of-fit test, these conditions must be met.
  - ✓ The observed frequencies must be obtained using a random sample.
  - ✓ Each expected frequency must be greater than or equal to 5.
- If these conditions are met, then the sampling distribution for the test is approximated by a chi-square distribution with $k - 1$ degrees of freedom, where k is the number of categories.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- The **test statistic** is

$$\chi^2 = \sum \left[\frac{(O - E)^2}{E} \right]$$

  where O represents the observed frequency of each category and E represents the expected frequency of each category.

- When the observed frequencies closely match the expected frequencies, the differences between O and E will be small and the chi-square test statistic will be close to 0. As such, the null hypothesis is unlikely to be rejected.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

- When there are large discrepancies (فروقات) between the observed frequencies and the expected frequencies, the differences between O and E will be large, resulting in a large chi-square test statistic.
- A large chi-square test statistic is evidence for rejecting the null hypothesis.
- So, the chi-square goodness-of-fit test is always a right-tailed test.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Performing a Chi-Square Goodness-of-Fit Test

IN WORDS	IN SYMBOLS
1. Verify that the observed frequencies were obtained from a random sample and each expected frequency is at least 5.	
2. Identify the claim. State the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= k - 1$
5. Determine the critical value.	Use Table 6 in Appendix B.
6. Determine the rejection region.	

THE CHI-SQUARE GOODNESS-OF-FIT TEST

7. Find the test statistic and sketch the sampling distribution.	$\chi^2 = \Sigma \frac{(O - E)^2}{E}$
8. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

A retail trade (تجار التجزئة) association claims that the tax preparation methods of adults are distributed as shown in the table at the bottom right. A tax preparation company randomly selects 300 adults and asks them how they prepare their taxes. The results are shown in the table at the top corner. At $\alpha = 0.01$, test the association's claim.

Survey results (n = 300)	
Accountant	61
By hand	42
Computer software	112
Friend/family	29
Tax preparation service	56

Distribution of tax preparation methods	
Accountant	24%
By hand	20%
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THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

A retail trade (تجار التجزئة) association claims that the tax preparation methods of adults are distributed as shown in the table at the bottom right. A tax preparation company randomly selects 300 adults and asks them how they prepare their taxes. The results are shown in the table at the top corner. At $\alpha = 0.01$, test the association's claim.

Tax preparation method	Observed frequency	Expected frequency
Accountant	61	72
By hand	42	60
Computer software	112	105
Friend/ family	29	18
Tax preparation service	56	45

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

H_0: The expected distribution of tax preparation methods is 24% by accountant, 20% by hand, 35% by computer software, 6% by friend or family, and 15% by tax preparation service. (**Claim**)

H_a: The distribution of tax preparation methods differs from the expected distribution.

Tax preparation method	Observed frequency	Expected frequency
Accountant	61	72
By hand	42	60
Computer software	112	105
Friend/ family	29	18
Tax preparation service	56	45

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

With the observed and expected frequencies, the chi-square **test statistic** is

$$\chi^2 = \Sigma\frac{(O - E)^2}{E}$$

$$= \frac{(61 - 72)^2}{72} + \frac{(42 - 60)^2}{60} + \frac{(112 - 105)^2}{105} + \frac{(29 - 18)^2}{18} + \frac{(56 - 45)^2}{45}$$

$$\approx 16.958.$$

Tax preparation method	Observed frequency	Expected frequency
Accountant	61	72
By hand	42	60
Computer software	112	105
Friend/ family	29	18
Tax preparation service	56	45

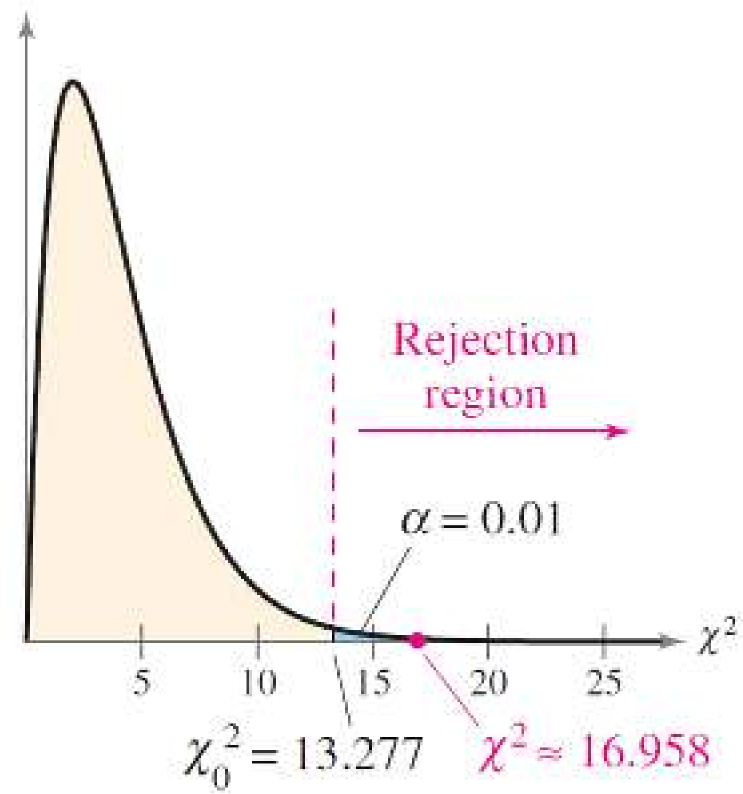
THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example The chi-square **test statistic** is 16.958.

- Because there are 5 categories, the chi-square distribution has

  $$\text{d.f.} = k - 1 = 5 - 1 = 4$$

  degrees of freedom.
- With $\text{d.f.} = 4$ and $\alpha = 0.01$, the critical value is $\chi_0^2 = 13.277$.



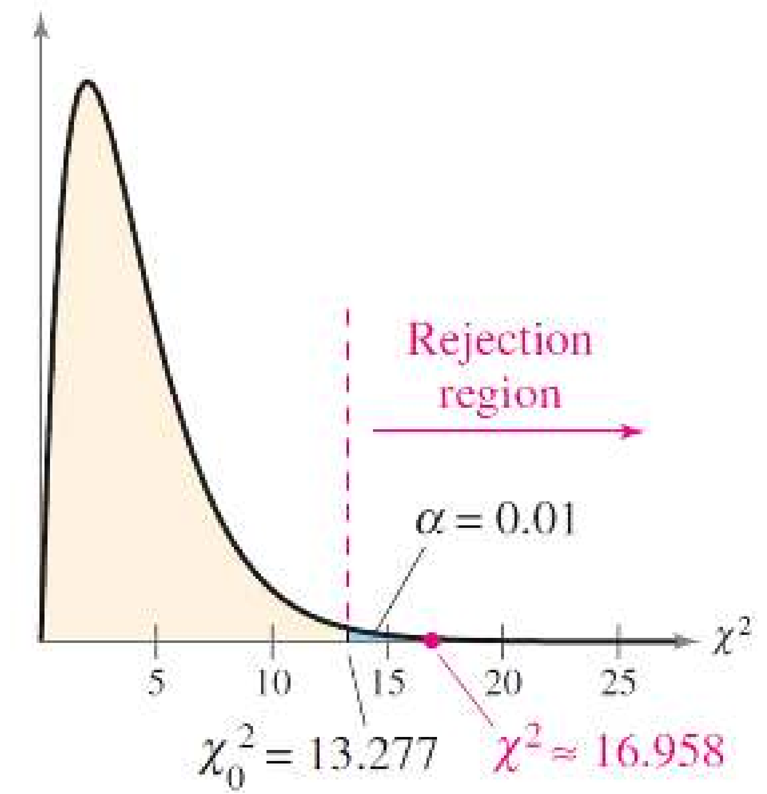
THE CHI-SQUARE GOODNESS-OF-FIT TEST

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

- Because χ^2 is in the rejection region, you reject the null hypothesis.
- There is enough evidence at the 1% level of significance to reject the claim that the distribution of tax preparation methods and the association's expected distribution are the same.



THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example

A researcher claims that the number of different-colored candies in bags of dark chocolate M&M's® is uniformly distributed. To test this claim, you randomly select a bag that contains 500 dark chocolate M&M's®. The results are shown in the table. At $\alpha = 0.10$, test the researcher's claim.

Color	Frequency, f
Brown	80
Yellow	95
Red	88
Blue	83
Orange	76
Green	78

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example $n = 500\,,\alpha\; = 0.10$

Color	p	O	E
Brown	$1/6$	80	$500/6 \approx 83.33$
Yellow	$1/6$	95	$500/6 \approx 83.33$
Red	$1/6$	88	$500/6 \approx 83.33$
Blue	$1/6$	83	$500/6 \approx 83.33$
Orange	$1/6$	76	$500/6 \approx 83.33$
Green	$1/6$	78	$500/6 \approx 83.33$

Color	Frequency, f
Brown	80
Yellow	95
Red	88
Blue	83
Orange	76
Green	78

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example $n = 500\ , \alpha\ = 0.10$

H_0: The expected distribution of the different-colored candies in bags of dark chocolate M&M's® is uniform. (**Claim**)
H_a: The distribution of the different-colored candies in bags of dark chocolate M&M's® is not uniform.

O	E
80	83.33
95	83.33
88	83.33
83	83.33
76	83.33
78	83.33

Because there are $k = 6$ categories, the chi-square distribution has d.f. $= k - 1 = 6 - 1 = 5$ degrees of freedom.

Using d.f. $= 5$ and $\alpha = 0.10$, the critical value is $\chi_0^2 = 9.236$.

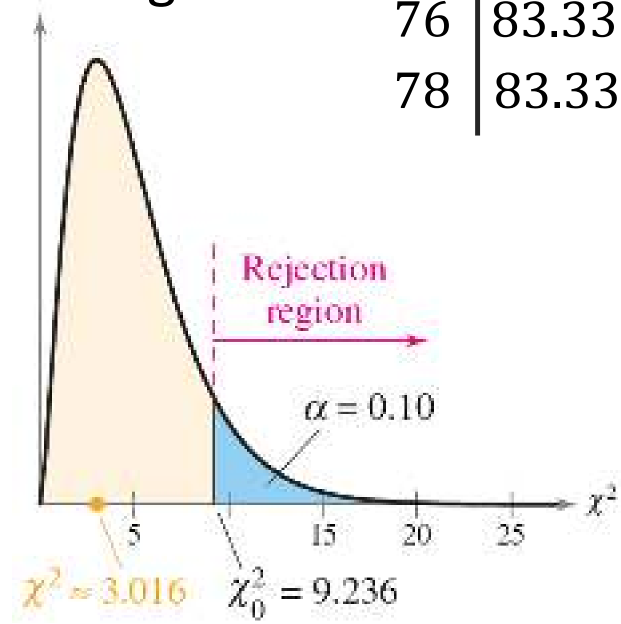
THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example $n = 500 \, , \alpha \ = 0.10$

H_0: The expected distribution of the different-colored candies in bags of dark chocolate M&M's® is uniform. (**Claim**)

H_a: The distribution of the different-colored candies in bags of dark chocolate M&M's® is not uniform.

O	E
80	83.33
95	83.33
88	83.33
83	83.33
76	83.33
78	83.33

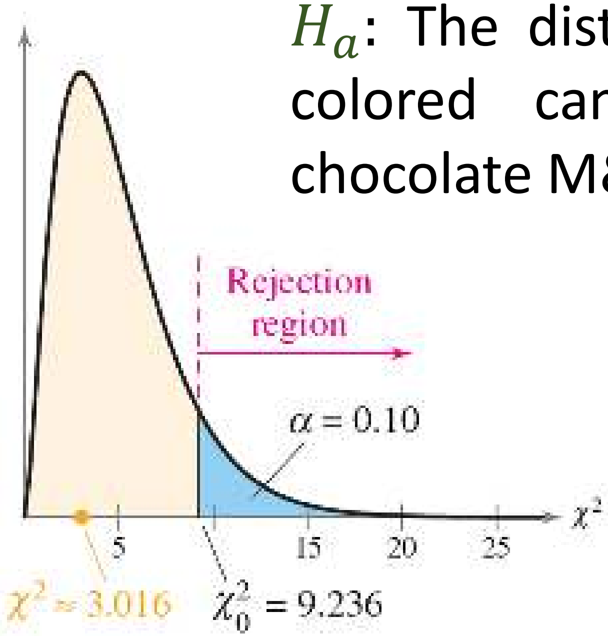


THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example H_0: The expected distribution of the different-colored candies in bags of dark chocolate M&M's® is uniform. (**Claim**)

H_a: The distribution of the different-colored candies in bags of dark chocolate M&M's® is not uniform.

O	E	$\frac{(O-E)^2}{E}$
80	83.33	0.1330721229
95	83.33	1.6343321733
88	83.33	0.2617172687
83	83.33	0.0013068523
76	83.33	0.6447725909
78	83.33	0.3409204368



The chi-square test statistic is 3.016

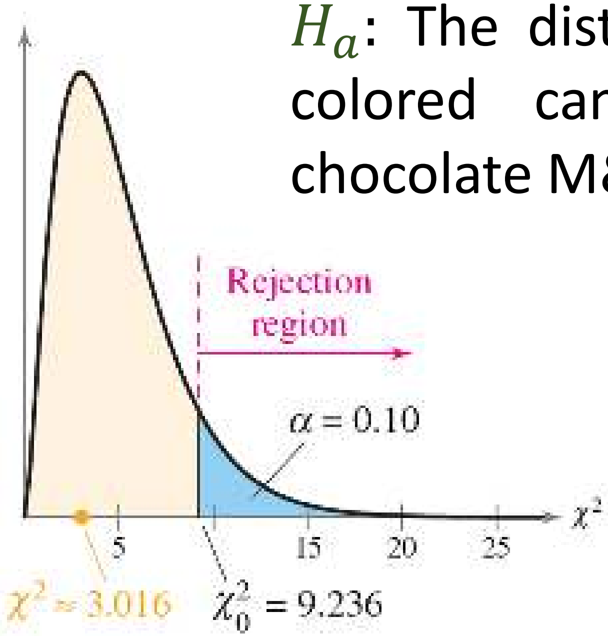
Because χ^2 is not in the rejection region, you fail to reject the null hypothesis.

THE CHI-SQUARE GOODNESS-OF-FIT TEST

Example H_0: The expected distribution of the different-colored candies in bags of dark chocolate M&M's® is uniform. (**Claim**)

H_a: The distribution of the different-colored candies in bags of dark chocolate M&M's® is not uniform.

O	E	$\frac{(O-E)^2}{E}$
80	83.33	0.1330721229
95	83.33	1.6343321733
88	83.33	0.2617172687
83	83.33	0.0013068523
76	83.33	0.6447725909
78	83.33	0.3409204368



The chi-square test statistic is 3.016

There is not enough evidence at the 10% level of significance to reject the claim that the distribution of the different-colored candies in bags of dark chocolate M&M's® is uniform.

Course: Biostatistics

Lecture No: [22]

Chapter: [10]

Chi-Square Tests and the F −Distribution

Section: [10.2]

Independence

CONTINGENCY TABLES

- You learned that two events are *independent* when the occurrence of one event does not affect the probability of the occurrence of the other event.
- But, suppose a medical researcher wants to determine whether there is a relationship between caffeine consumption (تناول الكافيين) and heart attack risk (خطر النوبات القلبية). Are these variables independent or are they dependent?
- In this section, you will learn how to use the chi-square test for independence to answer such a question.
- To perform a chi-square test for independence, you will use sample data that are organized in a **contingency table**.

CONTINGENCY TABLES

- **Definition**

An $r \times c$ **contingency table** shows the observed frequencies for two variables. The observed frequencies are arranged in r rows and c columns. The intersection of a row and a column is called a **cell**.

- **Example**

	Favorite way to eat ice cream				
Gender	**Cup**	**Cone**	**Sundae**	**Sandwich**	**Other**
Male	592	300	204	24	80
Female	410	335	180	20	55

FINDING THE EXPECTED FREQUENCY FOR CONTINGENCY TABLE CELLS

- The **expected frequency for a cell** $E_{r,c}$ in a contingency table is

$$E_{r,c} = \frac{(\text{sum of row } r)(\text{sum of column } c)}{\text{sample size}}$$

- When you find the sum of each row and column in a contingency table, you are calculating the *marginal frequencies*.
- A **marginal frequency** is the frequency that an entire category of one of the variables occurs.
- The observed frequencies in the interior of a contingency table are called **joint frequencies**.

FINDING THE EXPECTED FREQUENCY FOR CONTINGENCY TABLE CELLS

Example

Find the expected frequency for each cell in the contingency table. *Assume that the variables favorite way to eat ice cream and gender are independent*.

	Favorite way to eat ice cream				
Gender	**Cup**	**Cone**	**Sundae**	**Sandwich**	**Other**
Male	592	300	204	24	80
Female	410	335	180	20	55

FINDING THE EXPECTED FREQUENCY FOR CONTINGENCY TABLE CELLS

Example

Find the expected frequency for each cell in the contingency table. *Assume that the variables favorite way to eat ice cream and gender are independent.*

	Favorite way to eat ice cream					
Gender	**Cup**	**Cone**	**Sundae**	**Sandwich**	**Other**	**Total**
Male	592	300	204	24	80	1200
Female	410	335	180	20	55	1000
Total	1002	635	384	44	135	2200

$$E_{1,1} = \frac{1200 \cdot 1002}{2200} \approx 546.55$$

$$E_{1,2} = \frac{1200 \cdot 635}{2200} \approx 346.36$$

$$E_{1,3} = \frac{1200 \cdot 384}{2200} \approx 209.45$$

$$E_{1,4} = \frac{1200 \cdot 44}{2200} = 24$$

$$E_{1,5} = \frac{1200 \cdot 135}{2200} \approx 73.64$$

$$E_{2,1} = \frac{1000 \cdot 1002}{2200} \approx 455.45$$

$$E_{2,2} = \frac{1000 \cdot 635}{2200} \approx 288.64$$

$$E_{2,3} = \frac{1000 \cdot 384}{2200} \approx 174.55$$

$$E_{2,4} = \frac{1000 \cdot 44}{2200} = 20$$

$$E_{2,5} = \frac{1000 \cdot 135}{2200} \approx 61.36$$

THE CHI-SQUARE INDEPENDENCE TEST

- After finding the expected frequencies, you can test whether the variables are independent using a *chi-square independence test*.
- **Definition:**

  A **chi-square independence test** is used to test the independence of two variables. Using this test, you can determine whether the occurrence of one variable affects the probability of the occurrence of the other variable.
- Before performing a chi-square independence test, you must verify that
  - ✓ the observed frequencies were obtained from a random sample and
  - ✓ each expected frequency is at least 5.

THE CHI-SQUARE INDEPENDENCE TEST

- If these conditions are met, then the sampling distribution for the test is approximated by a chi-square distribution with

$$\text{d.f.} = (r-1)(c-1)$$

  degrees of freedom, where r and c are the number of rows and columns, respectively, of a contingency table.

- The **test statistic** is

$$\chi^2 = \sum \left[\frac{(O-E)^2}{E}\right]$$

  where O represents the observed frequencies and E represents the expected frequencies.

THE CHI-SQUARE INDEPENDENCE TEST

- To begin the independence test, you must first state a null hypothesis and an alternative hypothesis.
- For a chi-square independence test, the null and alternative hypotheses are

$$H_0: \quad \text{The variables are independent.}$$
$$H_a: \quad \text{The variables are dependent.}$$

- A large chi-square test statistic is evidence for rejecting the null hypothesis.
- So, the chi-square independence test is always a *right-tailed test*.

THE CHI-SQUARE INDEPENDENCE TEST

Performing a Chi-Square Independence Test

IN WORDS	IN SYMBOLS
1. Verify that the observed frequencies were obtained from a random sample and each expected frequency is at least 5.	
2. Identify the claim. State the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the degrees of freedom.	d.f. $= (r-1)(c-1)$
5. Determine the critical value.	Use Table 6 in Appendix B.

THE CHI-SQUARE INDEPENDENCE TEST

6. Determine the rejection region.	
7. Find the test statistic and sketch the sampling distribution.	$\chi^2 = \Sigma\frac{(O - E)^2}{E}$
8. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

THE CHI-SQUARE INDEPENDENCE TEST

Example

The contingency table shows the results of a random sample of 2200 adults classified by their favorite way to eat ice cream and gender. The expected frequencies are displayed in parentheses. At $\alpha = 0.01$, can you conclude that the variables favorite way to eat ice cream and gender are related?

	Favorite way to eat ice cream					
Gender	**Cup**	**Cone**	**Sundae**	**Sandwich**	**Other**	**Total**
Male	592 (546.55)	300 (346.36)	204 (209.45)	24 (24)	80 (73.64)	1200
Female	410 (455.45)	335 (288.64)	180 (174.55)	20 (20)	55 (61.36)	1000
Total	1002	635	384	44	135	2200

H_0: The variables favorite way to eat ice cream and gender are independent

H_a: The variables favorite way to eat ice cream and gender are dependent (Claim)

THE CHI-SQUARE INDEPENDENCE TEST

Example $\alpha = 0.01$

O	E
592	546.55
300	346.36
204	209.45
24	24
80	73.64
410	455.45
335	288.64
180	174.55
20	20
55	61.36

	Favorite way to eat ice cream					
Gender	**Cup**	**Cone**	**Sundae**	**Sandwich**	**Other**	**Total**
Male	592 (546.55)	300 (346.36)	204 (209.45)	24 (24)	80 (73.64)	1200
Female	410 (455.45)	335 (288.64)	180 (174.55)	20 (20)	55 (61.36)	1000
Total	1002	635	384	44	135	2200

H_0: The variables favorite way to eat ice cream and gender are independent

H_a: The variables favorite way to eat ice cream and gender are dependent (Claim)

THE CHI-SQUARE INDEPENDENCE TEST

Example $\alpha = 0.01$

O	E	$O - E$
592	546.55	45.45
300	346.36	−46.36
204	209.45	−5.45
24	24	0
80	73.64	6.36
410	455.45	−45.45
335	288.64	46.36
180	174.55	5.45
20	20	0
55	61.36	−6.36

THE CHI-SQUARE INDEPENDENCE TEST

Example $\alpha = 0.01$

O	E	$O - E$	$(O - E)^2$
592	546.55	45.45	2065.7025
300	346.36	−46.36	2149.2496
204	209.45	−5.45	29.7025
24	24	0	0
80	73.64	6.36	40.4496
410	455.45	−45.45	2065.7025
335	288.64	46.36	2149.2496
180	174.55	5.45	29.7025
20	20	0	0
55	61.36	−6.36	40.4496

THE CHI-SQUARE INDEPENDENCE TEST

Example $\alpha = 0.01$

O	E	$O - E$	$(O - E)^2$	$\frac{(O - E)^2}{E}$
592	546.55	45.45	2065.7025	3.7795
300	346.36	−46.36	2149.2496	6.2052
204	209.45	−5.45	29.7025	0.1418
24	24	0	0	0
80	73.64	6.36	40.4496	0.5493
410	455.45	−45.45	2065.7025	4.5355
335	288.64	46.36	2149.2496	7.4461
180	174.55	5.45	29.7025	0.1702
20	20	0	0	0
55	61.36	−6.36	40.4496	0.6592

THE CHI-SQUARE INDEPENDENCE TEST

Example $\alpha = 0.01$

O	E	$O - E$	$(O - E)^2$	$\frac{(O - E)^2}{E}$
592	546.55	45.45	2065.7025	3.7795
300	346.36	−46.36	2149.2496	6.2052
204	209.45	−5.45	29.7025	0.1418
24	24	0	0	0
80	73.64	6.36	40.4496	0.5493
410	455.45	−45.45	2065.7025	4.5355
335	288.64	46.36	2149.2496	7.4461
180	174.55	5.45	29.7025	0.1702
20	20	0	0	0
55	61.36	−6.36	40.4496	0.6592
				$\chi^2 = \Sigma \frac{(O - E)^2}{E} \approx 23.487$

The value of the test statistics

THE CHI-SQUARE INDEPENDENCE TEST

Example $\alpha = 0.01$

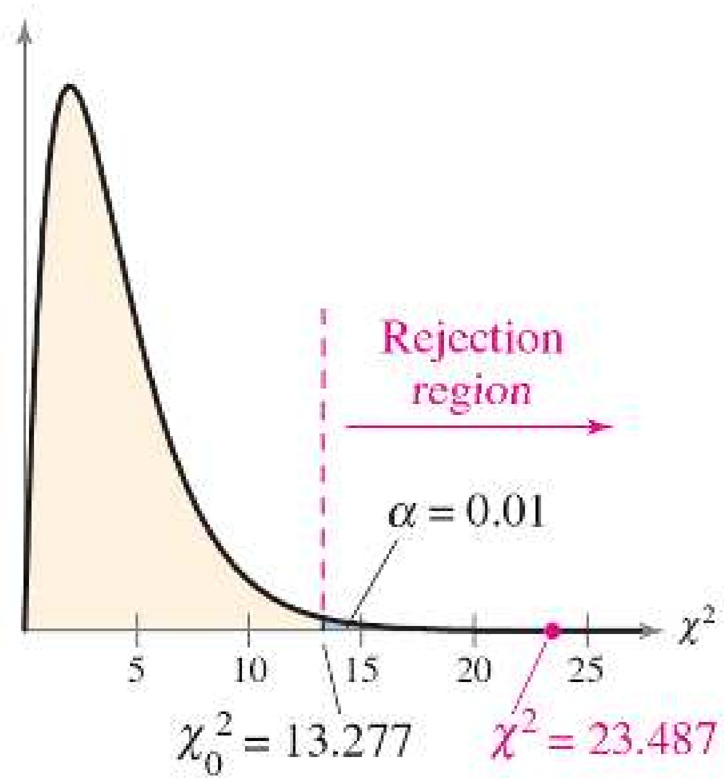
$$\chi^2 = 23.487$$

$$d.f. = (r-1)(c-1) = (2-1)(5-1)$$

Because $\chi^2 = 23.487$ is in the rejection region, you reject the null hypothesis.

There is enough evidence at the 1% level of significance to conclude that the variables favorite way to eat ice cream and gender are dependent.

Because d.f. $= 4$ and $\alpha = 0.01$, the critical value is $\chi_0^2 = 13.277$.



HOMOGENEITY OF PROPORTIONS TEST

- Another chi-square test that involves a contingency table is the *homogeneity of proportions test*.
- This test is used to determine whether several proportions are equal when samples are taken from different populations.
- Before the populations are sampled and the contingency table is made, the sample sizes are determined.
- After randomly sampling different populations, you can test whether the proportion of elements in a category is the same for each population using *the same guidelines as the a chi-square independence test*.

HOMOGENEITY OF PROPORTIONS TEST

- The null and alternative hypotheses are

  H_0: The proportions are equal.

  H_a: At least one of the proportions is different from the others.

- Performing a homogeneity of proportions test requires that:

  ✓ the observed frequencies be obtained using a random sample, and

  ✓ each expected frequency must be greater than or equal to 5.

- The chi-square homogeneity of proportions test is a *right-tailed* test.

HOMOGENEITY OF PROPORTIONS TEST

Question:

How the chi-square independence test is different from the chi-square homogeneity of proportions test.

Answer:

Both tests are very similar, but

- ✓ the chi-square test for independence tests whether the occurrence of one variable affects the probability of the occurrence of another variable,
- ✓ while the chi-square homogeneity of proportions test determines whether the proportions for categories from a population follow the same distribution as another population.

HOMOGENEITY OF PROPORTIONS TEST

Example

The contingency table shows the results of a random sample of patients with obsessive-compulsive disorder (اضطراب الوسواس القهري) after being treated with a drug or with a placebo. At $\alpha = 0.10$, perform a homogeneity of proportions test on the claim that the proportions of the results for drug and placebo treatments are the same.

	Treatment	
Result	**Drug**	**Placebo**
Improvement	39	25
No change	54	70

HOMOGENEITY OF PROPORTIONS TEST

Example

The contingency table shows the results of a random sample of patients with obsessive-compulsive disorder (اضطراب الوسواس القهري) after being treated with a drug or with a placebo. At $\alpha = 0.10$, perform a homogeneity of proportions test on the claim that the proportions of the results for drug and placebo treatments are the same.

The claim is:

"the proportions of the results for drug and placebo treatments are the same".

	Treatment		
Result	**Drug**	**Placebo**	**Total**
Improvement	39	25	64
No change	54	70	124
Total	93	95	188

$$E_{1,1} = \frac{(64)(93)}{188} \approx 31.66$$

$$E_{1,2} = \frac{(64)(95)}{188} \approx 32.34$$

$$E_{2,1} = \frac{(124)(93)}{188} \approx 61.34$$

$$E_{2,2} = \frac{(124)(95)}{188} \approx 62.66$$

HOMOGENEITY OF PROPORTIONS TEST

Example $\alpha = 0.10$

The claim is:
"the proportions of the results for drug and placebo treatments are the same".

H_0: The proportions are equal. (**Claim**)
H_a: At least one of the proportion is different from the others.

	Treatment		
Result	**Drug**	**Placebo**	**Total**
Improvement	39	25	64
No change	54	70	124
Total	93	95	188

$$E_{1,1} = \frac{(64)(93)}{188} \approx 31.66$$

$$E_{1,2} = \frac{(64)(95)}{188} \approx 32.34$$

$$E_{2,1} = \frac{(124)(93)}{188} \approx 61.34$$

$$E_{2,2} = \frac{(124)(95)}{188} \approx 62.66$$

HOMOGENEITY OF PROPORTIONS TEST

Example $\alpha = 0.10$

H_0: The proportions are equal. (**Claim**)
H_a: At least one of the proportion is different from the others.

	Treatment		
Result	**Drug**	**Placebo**	**Total**
Improvement	39	25	64
No change	54	70	124
Total	93	95	188

$$E_{1,1} = \frac{(64)(93)}{188} \approx 31.66$$

$$E_{1,2} = \frac{(64)(95)}{188} \approx 32.34$$

$$E_{2,1} = \frac{(124)(93)}{188} \approx 61.34$$

$$E_{2,2} = \frac{(124)(95)}{188} \approx 62.66$$

O	E	$\frac{(O-E)^2}{E}$
39	31.66	1.7017
25	32.34	1.6659
54	61.34	0.8783
70	62.66	0.8598
		5.1057

Test Statistics is $\chi^2 = 5.1059$

HOMOGENEITY OF PROPORTIONS TEST

Example $\alpha = 0.10$

H_0: The proportions are equal. (**Claim**)
H_a: At least one of the proportion is different from the others.

Test Statistics is $\chi^2 = 5.1059$ $\chi_0^2 = 2.706$

$\text{d.f.} = (r-1)(c-1) = 1$

	Treatment		
Result	**Drug**	**Placebo**	**Total**
Improvement	39	25	64
No change	54	70	124
Total	93	95	188

O	E	$\frac{(O-E)^2}{E}$
39	31.66	1.7017
25	32.34	1.6659
54	61.34	0.8783
70	62.66	0.8598
		5.1057

Degrees of freedom	α					
	0.995	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**
1	—	—	0.001	0.004	0.016	2.706
2	0.010	0.020	0.051	0.103	0.211	4.605
3	0.072	0.115	0.216	0.352	0.584	6.251

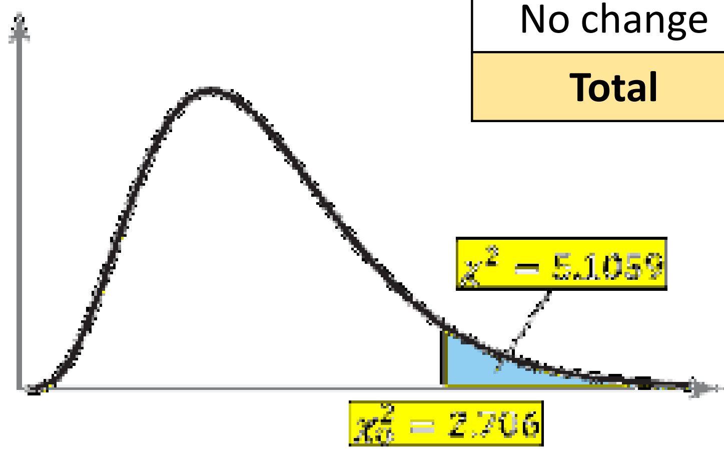
HOMOGENEITY OF PROPORTIONS TEST

Example $\alpha = 0.10$

H_0: The proportions are equal. (**Claim**)
H_a: At least one of the proportion is different from the others.

	Treatment		
Result	**Drug**	**Placebo**	**Total**
Improvement	39	25	64
No change	54	70	124
Total	93	95	188

Reject H_0



O	E	$\frac{(O-E)^2}{E}$
39	31.66	1.7017
25	32.34	1.6659
54	61.34	0.8783
70	62.66	0.8598
		5.1057

There is enough evidence at the 10% level of significance to reject the claim that the proportions of the results for drug and placebo treatments or the same.

Course: Biostatistics Lecture No: [23]

Chapter: [10]

Chi-Square Tests and the F −Distribution

Section: [10.3]

Comparing Two Variances

THE F −DISTRIBUTION

- In Chapter 8, you learned how to perform hypothesis tests *to compare population means and population proportions*.
- Recall from **Section 8.2** that the t −test for the difference between two population means depends on whether the population variances are equal.
- To determine whether the population variances are equal, you can perform a two-sample F −test.
- In this section, you will learn about the **F −distribution** and how it can be used to compare two variances.

THE F –DISTRIBUTION

Definition

Let s_1^2 and s_2^2 represent the sample variances of two different populations. If both populations are normal and the population variances σ_1^2 and σ_2^2 are equal, then the sampling distribution of

$$F = \frac{s_1^2}{s_2^2}$$

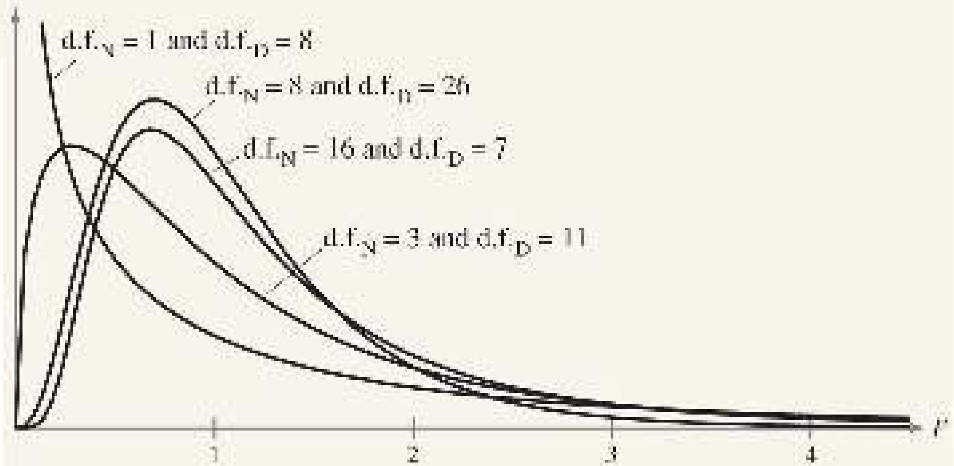
is an ***F*-distribution.**

THE F –DISTRIBUTION

Properties

Here are several properties of the F-distribution.

1. The F-distribution is a family of curves, each of which is determined by two types of degrees of freedom: the degrees of freedom corresponding to the variance in the numerator, denoted by **d.f.$_N$**, and the degrees of freedom corresponding to the variance in the denominator, denoted by **d.f.$_D$**.
2. The F-distribution is positively skewed and therefore the distribution is not symmetric (see figure below).



THE F −DISTRIBUTION

Properties

3. The total area under each F-distribution curve is equal to 1.
4. All values of F are greater than or equal to 0.
5. For all F-distributions, the mean value of F is approximately equal to 1.

Important Notes

- For unequal variances, designate the greater sample variance as s_1^2.
- So, in the sampling distribution of $F = \frac{s_1^2}{s_2^2}$, the variance in the numerator is greater than or equal to the variance in the denominator.
- This means that F is always greater than or equal to 1.
- As such, all one-tailed tests are right-tailed tests, and for all two-tailed tests, you need only to find the right-tailed critical value.

$d.f._D$: Degrees of freedom, denominator	$\alpha = 0.005$																		
	$d.f._N$: Degrees of freedom, numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	16211	20000	21615	22500	23056	23437	23715	23925	24091	24224	24426	24630	24836	24940	25044	25148	25253	25359	25465
2	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.5	199.5	199.5	199.5	199.5	199.5
3	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69	43.39	43.08	42.78	42.62	42.47	42.31	42.15	41.99	41.83
4	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97	20.70	20.44	20.17	20.03	19.89	19.75	19.61	19.47	19.32
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.38	13.15	12.90	12.78	12.66	12.53	12.40	12.27	12.14
6	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39	10.25	10.03	9.81	9.59	9.47	9.36	9.24	9.12	9.00	8.88
7	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51	8.38	8.18	7.97	7.75	7.65	7.53	7.42	7.31	7.19	7.08
8	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.01	6.81	6.61	6.50	6.40	6.29	6.18	6.06	5.95
9	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.23	6.03	5.83	5.73	5.62	5.52	5.41	5.30	5.19
10	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.66	5.47	5.27	5.17	5.07	4.97	4.86	4.75	4.64
11	12.73	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.24	5.05	4.86	4.76	4.65	4.55	4.44	4.34	4.23
12	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.91	4.72	4.53	4.43	4.33	4.23	4.12	4.01	3.90
13	11.37	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94	4.82	4.64	4.46	4.27	4.17	4.07	3.97	3.87	3.76	3.65
14	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72	4.60	4.43	4.25	4.06	3.96	3.86	3.76	3.66	3.55	3.44
15	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.25	4.07	3.88	3.79	3.69	3.58	3.48	3.37	3.26
16	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38	4.27	4.10	3.92	3.73	3.64	3.54	3.44	3.33	3.22	3.11
17	10.38	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25	4.14	3.97	3.79	3.61	3.51	3.41	3.31	3.21	3.10	2.98
18	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14	4.03	3.86	3.68	3.50	3.40	3.30	3.20	3.10	2.99	2.87
19	10.07	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04	3.93	3.76	3.59	3.40	3.31	3.21	3.11	3.00	2.89	2.78
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.68	3.50	3.32	3.22	3.12	3.02	2.92	2.81	2.69
21	9.83	6.89	5.73	5.09	4.68	4.39	4.18	4.01	3.88	3.77	3.60	3.43	3.24	3.15	3.05	2.95	2.84	2.73	2.61
22	9.73	6.81	5.65	5.02	4.61	4.32	4.11	3.94	3.81	3.70	3.54	3.36	3.18	3.08	2.98	2.88	2.77	2.66	2.55
23	9.63	6.73	5.58	4.95	4.54	4.26	4.05	3.88	3.75	3.64	3.47	3.30	3.12	3.02	2.92	2.82	2.71	2.60	2.48
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.42	3.25	3.06	2.97	2.87	2.77	2.66	2.55	2.43
25	9.48	6.60	5.46	4.84	4.43	4.15	3.94	3.78	3.64	3.54	3.37	3.20	3.01	2.92	2.82	2.72	2.61	2.50	2.38
26	9.41	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49	3.33	3.15	2.97	2.87	2.77	2.67	2.56	2.45	2.33
27	9.34	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45	3.28	3.11	2.93	2.83	2.73	2.63	2.52	2.41	2.25
28	9.28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41	3.25	3.07	2.89	2.79	2.69	2.59	2.48	2.37	2.29
29	9.23	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38	3.21	3.04	2.86	2.76	2.66	2.56	2.45	2.33	2.24
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.18	3.01	2.82	2.73	2.63	2.52	2.42	2.30	2.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	2.95	2.78	2.60	2.50	2.40	2.30	2.18	2.06	1.93
60	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.74	2.57	2.39	2.29	2.19	2.08	1.96	1.83	1.69
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.54	2.37	2.19	2.09	1.98	1.87	1.75	1.61	1.43
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.36	2.19	2.00	1.90	1.79	1.67	1.53	1.36	1.00

$d.f._D$: Degrees of freedom, denominator	$\alpha = 0.01$																		
	$d.f._N$: Degrees of freedom, numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	4.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

$d.f._D$: Degrees of freedom, denominator	$\alpha = 0.025$																		
	$d.f._N$: Degrees of freedom, numerator																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**15**	**20**	**24**	**30**	**40**	**60**	**120**	**∞**
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.98	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.25	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

$d.f._D$: Degrees of freedom, denominator	$\alpha = 0.05$																		
	$d.f._N$: Degrees of freedom, numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

d.f.$_D$: Degrees of freedom, denominator	$\alpha = 0.10$																		
	d.f.$_N$: Degrees of freedom, numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

THE F −DISTRIBUTION

Finding Critical Values for the F-Distribution

1. Specify the level of significance α.
2. Determine the degrees of freedom for the numerator $d.f._N$.
3. Determine the degrees of freedom for the denominator $d.f._D$.
4. Use Table 7 in Appendix B to find the critical value. When the hypothesis test is
   a. one-tailed, use the α F-table.
   b. two-tailed, use the $\frac{1}{2}\alpha$ F-table.

   Note that because F is always greater than or equal to 1, all one-tailed tests are right-tailed tests. For two-tailed tests, you need only to find the right-tailed critical value.

THE F –DISTRIBUTION

Example Find the critical F –value for a right-tailed test when $\alpha = 0.10$, $\text{d.f.}_N = 5$, and $\text{d.f.}_D = 28$.

$F_0 = 2.06$

d.f.$_D$: Degrees of freedom, denominator	$\alpha = 0.10$							
	d.f.$_N$: Degrees of freedom, numerator							
	1	2	3	4	5	6	7	8
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88

THE F −DISTRIBUTION

Example Find the critical F −value for a two-tailed test when $\alpha = 0.05$, $\text{d.f.}_N = 4$, and $\text{d.f.}_D = 8$.

$$\frac{1}{2}\alpha = \frac{1}{2}(0.05) = 0.025$$

$$F_0 = 5.05$$

d.f.$_D$: Degrees of freedom, denominator	$\alpha = 0.025$							
	d.f.$_N$: Degrees of freedom, numerator							
	1	2	3	4	5	6	7	8
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10

THE TWO-SAMPLE F –TEST FOR VARIANCES

A **two-sample F-test** is used to compare two population variances σ_1^2 and σ_2^2. To perform this test, these conditions must be met.

1. The samples must be random.
2. The samples must be independent.
3. Each population must have a normal distribution.

The **test statistic** is

$$F = \frac{s_1^2}{s_2^2}$$

where s_1^2 and s_2^2 represent the sample variances with $s_1^2 \geq s_2^2$. The numerator has $\text{d.f.}_N = n_1 - 1$ degrees of freedom and the denominator has $\text{d.f.}_D = n_2 - 1$ degrees of freedom, where n_1 is the size of the sample having variance s_1^2 and n_2 is the size of the sample having variance s_2^2.

THE TWO-SAMPLE F –TEST FOR VARIANCES

Using a Two-Sample F-Test to Compare σ_1^2 and σ_2^2

IN WORDS	IN SYMBOLS
1. Verify that the samples are random and independent, and the populations have normal distributions.	
2. Identify the claim. State the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom for the numerator and the denominator.	$\text{d.f.}_N = n_1 - 1$ $\text{d.f.}_D = n_2 - 1$

THE TWO-SAMPLE F –TEST FOR VARIANCES

5. Determine the critical value.	Use Table 7 in Appendix B.
6. Determine the rejection region.	
7. Find the test statistic and sketch the sampling distribution.	$F = \frac{s_1^2}{s_2^2}$
8. Make a decision to reject or fail to reject the null hypothesis.	If F is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

THE TWO-SAMPLE F –TEST FOR VARIANCES

Example A restaurant manager is designing a system that is intended to decrease the variance of the time customers wait before their meals are served. Under the old system, a random sample of 10 customers had a variance of 400. Under the new system, a random sample of 21 customers had a variance of 256. At $\alpha = 0.10$, is there enough evidence to convince the manager to switch to the new system? Assume both populations are normally distributed.

	Old System	New System
Sample Size	10	21
Variance	400	256

THE TWO-SAMPLE F −TEST FOR VARIANCES

Example A restaurant manager is designing a system that is intended to decrease the variance of the time customers wait before their meals are served. Under the old system, a random sample of 10 customers had a variance of 400. Under the new system, a random sample of 21 customers had a variance of 256. At $\alpha = 0.10$, is there enough evidence to convince the manager to switch to the new system? Assume both populations are normally distributed.

	Old System	New System
Sample Size	$n_1 = 10$	$n_2 = 21$
Variance	$s_1^2 = 400$	$s_2^2 = 256$

$$H_0: \quad \sigma_1^2 \leq \sigma_2^2$$
$$H_a: \quad \sigma_1^2 > \sigma_2^2 \quad \text{(Claim)}$$

THE TWO-SAMPLE F –TEST FOR VARIANCES

Example $\alpha = 0.10$

	Old System	New System
Sample Size	$n_1 = 10$	$n_2 = 21$
Variance	$s_1^2 = 400$	$s_2^2 = 256$

H_0: $\sigma_1^2 \leq \sigma_2^2$

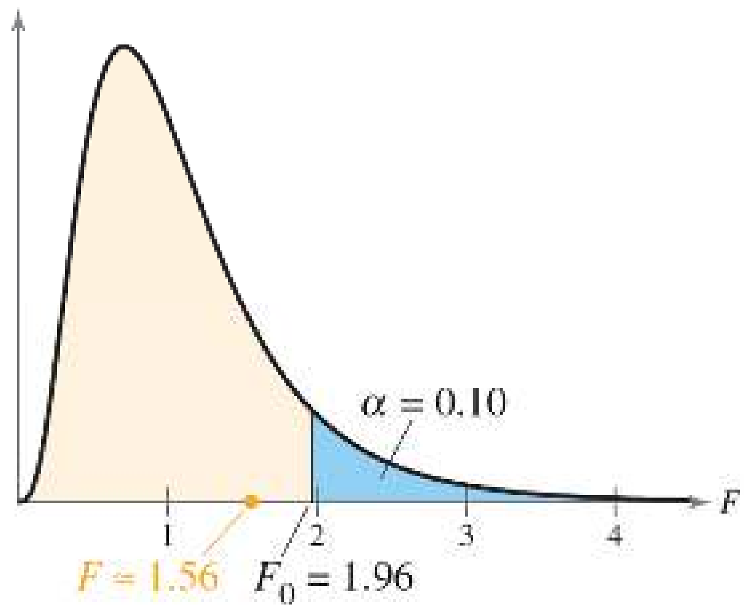
H_a: $\sigma_1^2 > \sigma_2^2$ (Claim)

$\text{d.f.}_\text{N} = n_1 - 1 = 9$

$\text{d.f.}_\text{D} = n_2 - 1 = 20$

The critical value is $F_0 = 1.96$

The test statistics is $F = \dfrac{s_1^2}{s_2^2} = \dfrac{400}{256} \approx 1.56$



THE TWO-SAMPLE F –TEST FOR VARIANCES

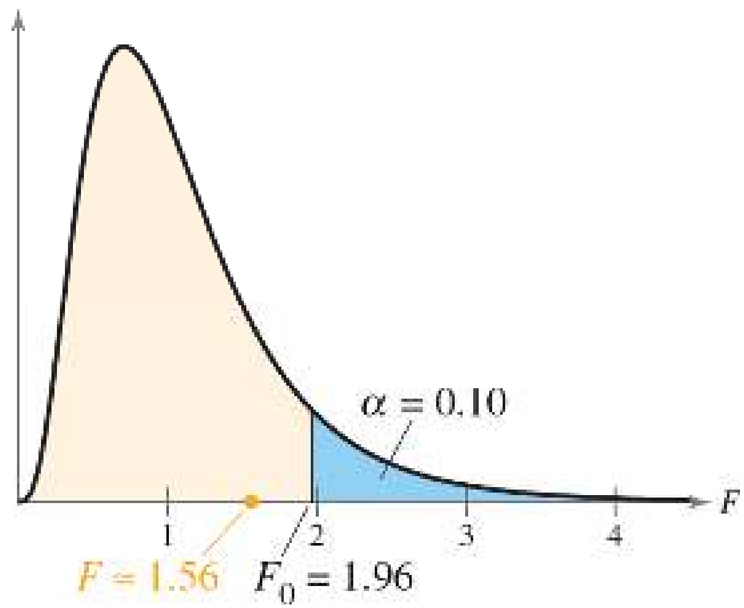
Example $\alpha = 0.10$

	Old System	New System
Sample Size	$n_1 = 10$	$n_2 = 21$
Variance	$s_1^2 = 400$	$s_2^2 = 256$

H_0: $\sigma_1^2 \leq \sigma_2^2$

H_a: $\sigma_1^2 > \sigma_2^2$ (Claim)

Because F is not in the rejection region, you fail to reject the null hypothesis.



There is not enough evidence at the 10% level of significance to convince the manager to switch to the new system.