CHAPTER (9)

SURFACE RESISTANCE

HOMEWORK (1)

9.6, 9.21, 9.37, 9.73

Problem (9.6)

$$u = \frac{y}{L} U_{\max}$$

$$\tau_s = \mu \frac{u}{L} = \frac{F_s}{A}$$

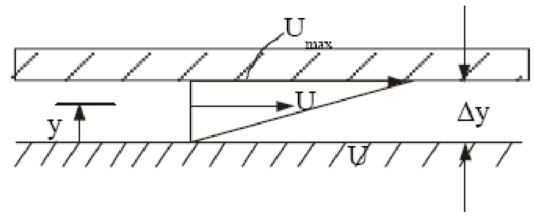
PROBLEM 9.6

Situation: A plate being pulled over oil is described in the problem statement.

Find: (a) Express the velocity mathematically in terms of the coordinate system shown.
(b) Whether flow is rotation or irrotational.
(c) Whether continuity is satisfied.
(d) Force required to produce plate motion.

ANALYSIS

By similar triangles $u/y = u_{\max}/\Delta t$



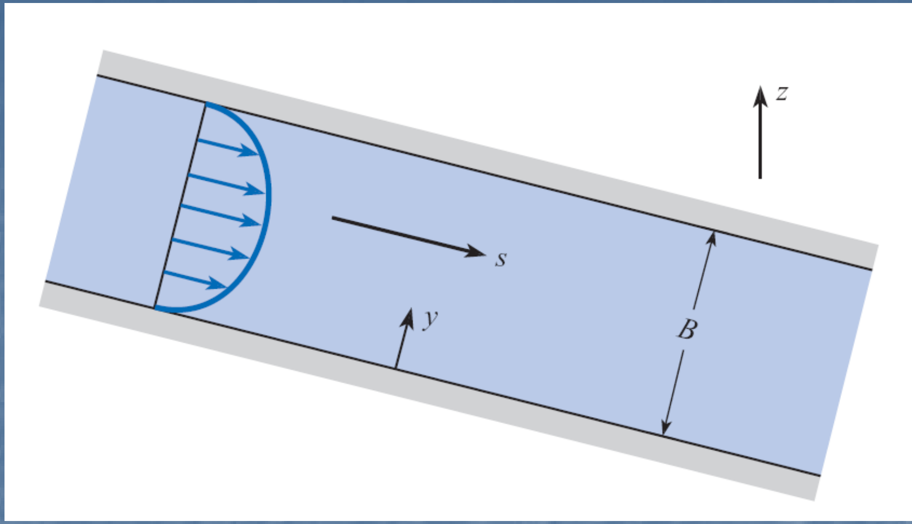
or

$$\begin{aligned} u &= (u_{\max}/\Delta y)y \\ u &= (0.3/0.002)y \text{ m/s} \\ & \boxed{u = 150 \text{ y m/s}} \\ v &= 0 \end{aligned}$$

For flow to be irrotational $\partial u/\partial y = \partial V/\partial x$ here $\partial u/\partial y = 150$ and $\partial V/\partial x = 0$. The equation is not satisfied; flow is rotational .
$\partial u/\partial x + \partial v/\partial y = 0$ (continuity equation) $\partial u/\partial x = 0$ and $\partial v/\partial y = 0$
so continuity is satisfied.
Use the same formula as developed for solution to Prob. 9-1, but $W \sin\theta = F_{shear}$. Then

$$\begin{aligned} F_s &= A\mu V/t \\ F_s &= 0.3 \times (1 \times 0.3) \times 4/0.002 \\ & \boxed{F_s = 180 \text{ N}} \end{aligned}$$

Problem (9.21)



Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Shear (drag) force on lower plate.

ANALYSIS

$$u = -(\gamma/2\mu)(By - y^2)dh/ds$$

u_{max} occurs at $y = B/2$ so

$$u_{max} = -(\gamma/2\mu)(B^2/2 - B^2/4)dh/ds = -(\gamma/2\mu)(B^2/4)dh/ds$$

From problem statement $dp/ds = -1200$ Pa/m and $dh/ds = (1/\gamma)dp/ds$. Also $B = 2$ mm= 0.002 m and $\mu = 10^{-1}$N·s/m^2. Then

$$\begin{aligned} u_{\max} &= -(\gamma/2\mu)(B^2/4)((1/\gamma)(-1,200)) \\ &= (B^2/8\mu)(1,200) \\ &= (0.002^2/(8 \times 0.1))(1,200) \\ &= 0.006 \text{ m/s} \end{aligned}$$

$$\boxed{u_{\max} = 6.0 \text{ mm/s}}$$

$$\begin{aligned} F_s &= \tau A = \mu(du/dy) \times 2 \times 1.5 \\ \tau &= \mu \times [-(\gamma/2\mu)(B - 2y)dh/ds] \end{aligned}$$

but τ_{plate} occurs at $y = 0$. So

$$\begin{aligned} F_s &= -\mu \times (\gamma/2\mu) \times B \times (-1,200/\gamma) \times 3 = (B/2) \times 1,200 \times 3 \\ &= (0.002/2) \times 1,200 \times 3 \end{aligned}$$

$$\boxed{F_s = 3.6 \text{ N}}$$

Problem (9.37)

$$\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}}$$

$$\tau_0 = 0.332\mu \frac{U_0}{x}\sqrt{\mathrm{Re}_x}$$

Situation: A thin plate is held stationary in a stream of water–additional details are provided in the problem statement.

Find: (a) Thickness of boundary layer.
(b) Distance from leading edge.
(c) Shear stress.

APPROACH

Find Reynolds number. Then, calcuate the boundary layer thickness and shear stress with the appropriate correlations

Problem (9.37)

$$\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}}$$

$$\tau_0 = 0.332\mu \frac{U_0}{x}\sqrt{\mathrm{Re}_x}$$

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re} &= U_0 x/\nu \\ x &= \mathrm{Re}\,\nu/U_0 \\ &= 500,000 \times 1.22 \times 10^{-5}/5 \end{aligned}$$

$$\boxed{x = 1.22 \text{ ft}}$$

Boundary layer thickness correlation

$$\begin{aligned} \delta &= 5x/\mathrm{Re}_x^{1/2} \quad \text{(laminar flow)} \\ &= 5 \times 1.22/(500,000)^{1/2} \\ &= 0.0086 \text{ ft} \end{aligned}$$

$$\boxed{\delta = 0.103 \text{ in.}}$$

Local shear stress correlation

$$\begin{aligned} \tau_0 &= 0.332\mu(U_0/x)\,\mathrm{Re}_x^{1/2} \\ &= 0.332 \times 2.36 \times 10^{-5}(5/1.22) \times (500,000)^{1/2} \end{aligned}$$

$$\boxed{\tau_0 = 0.0227 \text{ lbf/ft}^2}$$

Problem (9.73)

PROBLEM 9.73

Situation: A boundary layer next to the smooth hull of a ship is described in the problem statement.

Find: (a) Thickness of boundary layer at $x = 100\,\mathrm{ft}$.
(b) Velocity of water at $y/\delta = 0.5$.
(c) Shear stress on hull at $x = 100\,\mathrm{ft}$.

Properties: Table A.5 (water at 60 °F): $\rho = 1.94\,\mathrm{slug/\,ft^3}$, $\gamma = 62.37\,\mathrm{lbf/\,ft^3}$, $\mu = 2.36 \times 10^{-5}\,\mathrm{lbf \cdot s/\,ft^2}$, $\nu = 1.22 \times 10^{-5}\,\mathrm{ft^2/\,s}$.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_x &= \frac{Ux}{\nu} \\ &= \frac{(45)(100)}{1.22 \times 10^{-5}} = 3.689 \times 10^8 \end{aligned}$$

Local shear stress coefficient

$$\begin{aligned} c_f &= \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} = \frac{0.455}{\ln^2(0.06 * 3.689 \times 10^8)} \\ &= 0.001591 \end{aligned}$$

Local shear stress

$$\tau_0 = c_f \left(\frac{\rho U_0^2}{2} \right)$$
$$= (0.001591) \left(\frac{1.94 \times 45^2}{2} \right)$$

$$\boxed{\tau_0 = 3.13 \text{ lbf/ft}^2 \text{ (c)}}$$

Shear velocity

$$u_* = (\tau_0/\rho)^{0.5}$$
$$= (3.13/1.94)^{0.5}$$
$$= 1.270 \text{ ft/s}$$

Boundary layer thickness (turbulent flow)

$$\begin{aligned}\delta/x &= 0.16\,\mathrm{Re}_x^{-1/7} = 0.16\left(3.689\times 10^8\right)^{-1/7} \\ &= 0.009556 \\ \delta &= (0.009556)(100)\end{aligned}$$

$$\boxed{\delta = 0.956 \text{ ft (a)}}$$

$$\delta/2 = 0.48 \text{ ft}$$

From Fig. 9-12 at $y/\delta = 0.50$, $(U_0 - u)/u_* \approx 3$ Then

$$(45 - u)/1.27 = 3$$

$$\boxed{u\,(y = \delta/2) = 41.2 \text{ ft/s (b)}}$$

THE END