CHAPTER (2)

FLUID PROPERTIES

HOMEWORK (1)

2.11, 2.20, 2.33,2.54

Problem (2.11) Ideal Gas Law

PROBLEM 2.11

Situation: The application is a helium filled balloon of radius $r = 1.3$ m.
$p = 0.89$ bar $= 89$ kPa.
$T = 22\,^\circ\text{C} = 295.2$ K.

Find: Weight of helium inside balloon.

Properties: From Table A.2, $R_{He} = 2077$ J/kg·K.

APPROACH

Weight is given by $W = mg$. Mass is related to volume by $m = \rho *$Volume. Density can be found using the ideal gas law.

ANALYSIS

Volume in a sphere

$$\begin{aligned} \text{Volume} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi 1.3^3 \text{ m}^3 \\ &= 9.203 \text{ m}^3 \end{aligned}$$

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{89,000\,\text{N/m}^2}{(2077\,\text{J/kg}\cdot\text{K})\,(295.2\,\text{K})} \\ &= 0.145\,\text{kg/m}^3\end{aligned}$$

Weight of helium

$$\begin{aligned}W &= \rho \times \text{Volume} \times g \\ &= \left(0.145\,\text{kg/m}^3\right) \times \left(9.203\,\text{m}^3\right) \times \left(9.81\,\text{m/s}^2\right) \\ &= 13.10\,\text{N}\end{aligned}$$

$$\boxed{\text{Weight} = 13.1\ \text{N}}$$

Problem (2.20)

PROBLEM 2.20

Situation: Kinematic viscosity of methane at 15°C and 1 atm is $1.59 \times 10^{-5}\ \mathrm{m^2/s}$.

Find: Kinematic viscosity of methane at 200°C and 2 atm.

Properties: From Table A.2, $S = 198$ K.

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0 + S}{T + S}\right)$$

Problem (2.20)

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0 + S}{T + S}\right)$$

$$\nu = \frac{\mu}{\rho}$$

$$\rho = \frac{p}{RT}$$

APPROACH

Apply the ideal gas law and Sutherland's equation.

ANALYSIS

$$\nu = \frac{\mu}{\rho}$$

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{\rho_o}{\rho}$$

Ideal-gas law

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{p_o}{p}\frac{T}{T_o}$$

Sutherland's equation

$$\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S}$$

so

$$\frac{\nu}{\nu_o} = \frac{1}{2}\left(\frac{473}{288}\right)^{5/2}\frac{288 + 198}{473 + 198} = 1.252$$

and

$$\nu = 1.252 \times 1.59 \times 10^{-5}\ \mathrm{m^2/s} = \boxed{1.99 \times 10^{-5}\ \mathrm{m^2/s}}$$

Problem (2.33)

PROBLEM 2.33

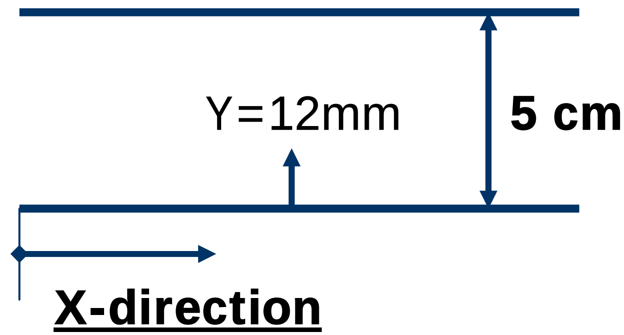
Situation: Glycerin is flowing in between two stationary plates. The plate spacing is $B = 5\,\text{cm}$.

The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)$$

where the pressure gradient is $dp/dx = -1.6\,\text{kN/m}^3$

Pressure gradient



Find:

a.) Velocity and shear stress at12 mm from wall (i.e. at $y = 12\,\text{mm}$).

b.) Velocity and shear stress at the wall (i.e. at $y = 0\,\text{mm}$).

Properties: Glycerin at 20 °C from Table A.4: $\mu = 1.41\,\text{N}\cdot\text{s/m}^2$.

APPROACH

Find velocity by direct substitution into the specified velocity distribution. Find shear stress using $\tau = \mu\,(du/dy)$, where the rate-of-strain (i.e. the derivative du/dy) is found by differentiating the velocity distribution.

ANALYSIS

a.) Velocity (at $y = 12\,\mathrm{mm}$)

$$
\begin{aligned}
u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\
&= -\frac{1}{2\left(1.41\,\mathrm{N\cdot s/\,m^2}\right)}\left(-1600\,\mathrm{N/\,m^3}\right)\left(\left(0.05\,\mathrm{m}\right)\left(0.012\,\mathrm{m}\right) - \left(0.012\,\mathrm{m}\right)^2\right) \\
&= 0.2587\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}
$$

$$
\boxed{u\left(y = 12\,\mathrm{mm}\right) = 0.259\,\mathrm{m/\,s}}
$$

Rate of strain (general expression)

$$
\begin{aligned}
\frac{du}{dy} &= \frac{d}{dy}\left(-\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\right) \\
&= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)\frac{d}{dy}\left(By - y^2\right) \\
&= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)\left(B - 2y\right)
\end{aligned}
$$

Rate of strain (at $y = 12\,\text{mm}$)

$$\begin{aligned}
\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \\
&= \left(-\frac{1}{2\,(1.41\,\text{N}\cdot\text{s}/\,\text{m}^2)}\right)\left(-1600\frac{\text{N}}{\text{m}^3}\right)(0.05\,\text{m} - 2\times 0.012\,\text{m}) \\
&= 14.75\,\text{s}^{-1}
\end{aligned}$$

Shear stress

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \left(1.41\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)(14.75\,\text{s}^{-1}) \\
&= 20.798\,\text{Pa}
\end{aligned}$$

$$\boxed{\tau\,(y = 12\,\text{mm}) = 20.8\,\text{Pa}}$$

b.) Velocity (at $y = 0\,\text{mm}$)

$$\begin{aligned}
u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\
&= -\frac{1}{2\,(1.41\,\text{N}\cdot\text{s}/\,\text{m}^2)}\left(-1600\,\text{N}/\,\text{m}^3\right)\left((0.05\,\text{m})\,(0\,\text{m}) - (0\,\text{m})^2\right) \\
&= 0.00\frac{\text{m}}{\text{s}}
\end{aligned}$$

$$\boxed{u\,(y = 0\,\mathrm{mm}) = 0\,\mathrm{m/\,s}}$$

Rate of strain (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}
\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \\
&= \left(-\frac{1}{2\,(1.41\,\mathrm{N\cdot s/\,m^2})}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m^3}}\right)(0.05\,\mathrm{m} - 2\times 0\,\mathrm{m}) \\
&= 28.37\,\mathrm{s^{-1}}
\end{aligned}$$

Shear stress (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \left(1.41\frac{\mathrm{N\cdot s}}{\mathrm{m^2}}\right)\left(28.37\,\mathrm{s^{-1}}\right) \\
&= 40.00\,\mathrm{Pa}
\end{aligned}$$

$$\boxed{\tau\,(y = 0\,\mathrm{mm}) = 40.0\,\mathrm{Pa}}$$

COMMENTS

1. As expected, the velocity at the wall (i.e. at $y = 0$) is zero due to the no slip condition.

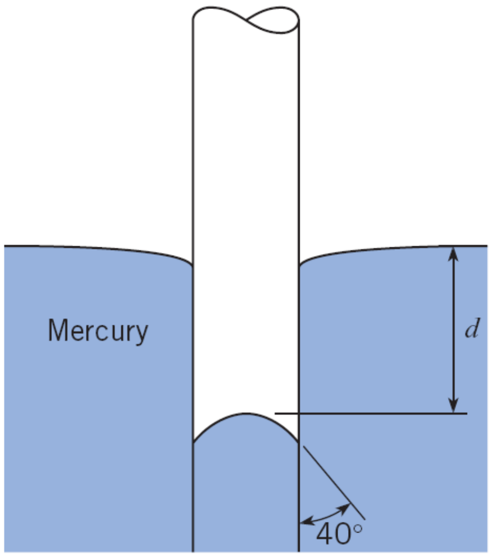
2. As expected, the shear stress at the wall is larger than the shear stress away from the wall. This is because shear stress is maximum at the wall and zero along the centerline (i.e. at $y = B/2$).

Problem (2.54)

PROBLEM 2.54

Situation: A glass tube is immersed in a pool of mercury—details are provided in the problem statement.

Find: Depression distance of mercury: d



APPROACH

Apply equilibrium and the surface tension force equation.

ANALYSIS

Surface tension=pressure forces

$$\sigma_S A_S = p A_{C.S}$$

$$\cos\theta \pi d \sigma = \Delta h \gamma \frac{\pi d^2}{4}$$

Solving for Δh results in

$$\Delta h = \frac{4 \cos\theta \sigma}{\gamma d}$$

Substitute in values

$$\begin{aligned} \Delta h &= \frac{4 \times \cos 40 \times 0.514}{(13.6 \times 9810) \times 0.001} \\ &= 0.0118\,\text{m} \end{aligned}$$

$$\boxed{\Delta h = 11.8\,\text{mm}}$$

<u>THE END</u>