7.1 INTRODUCTION
The ac-to-ac power converters available in industry today do not actually convert power directly from a.c. power of one frequency to a.c. power of another frequency. Instead, these converters first convert electrical power to d.c. using a rectifier, and then convert power back into a.c. using an inverter. These are called two-stage converters. However, a cycloconverter is a frequency changer that converts an a.c. supply of fixed input frequency directly to an a.c. output of another frequency. Cycloconverters not only eliminate the problem of having multiple systems to perform a single function, they also limit the flow of power to a single switch at any one period in time. Therefore, there is no bus link, d.c. or otherwise, included in a cycloconverter topology between power input and power output.

Cycloconverters are used in many industrial applications as one-stage frequency changer for a.c. motor drives and other high power, low speed devices such as gearless cement mills, steel rolling mills, ore grinding mills, pumps and compressors, and mine winders. Cycloconverters can be classified into two categories depending upon the method how the firing angle 'α' is controlled. These are:

1. Phase-controlled cycloconverters, in which the firing angle is controlled by adjustable gate pulses as in controlled rectifier circuits.
2. Envelope cycloconverters, in which the switches remain fully-on like diodes and conduct for consecutive half cycles. Depending upon the converter use for conversion, the cycloconverters may also be classified as,
   (i) 2-pulse cycloconverters.
   (ii) 3-pulse cycloconverters.
   (iii) 6-pulse cycloconverters.
Another classification is based on the input and output phases of the cycloconverters as,
   (i) Single-phase to single-phase cycloconverters.
   (ii) Three-phase to single-phase cycloconverters.
   (iii) Three-phase to three-phase cycloconverters

7.2 SINGLE-PHASE TO SINGLE-PHASE CYCLOCONVERTER
The basic circuit of the single-phase two-pulse cycloconverter is shown in Fig. 7.1. The converter consists of four thyristors and a center-tap transformer. By controlling the opening and closing of the thyristors, it is possible to fabricate output voltage waveforms having a fundamental component of the desired output frequency.

![Fig.7.1 Single-phase to single-phase two-pulse cycloconverter basic circuit.](image)

For example, if it is required to generate an output voltage wave of 25 Hz from an input voltage of 50 Hz, then in the positive half-cycle of the input voltage $v_{an}$, thyristor $T_1$ conducts from 0 to $\pi$, and in the positive half-cycle of the input voltage $v_{bn}$ $T_3$ conducts from $\pi$ to $2\pi$, hence output voltage is positive. Similarly, during the negative half-cycle of $v_{bn}$, $T_2$ conducts from $2\pi$ to $3\pi$ and during the negative half-cycle of $v_{an}$ $T_4$ conducts from $3\pi$ to $4\pi$, hence the load voltage becomes negative and the sequence is repeated. The waveforms of this converter are shown in
Fig. 7.2. The input supply voltage is shown in Fig. 7.2 (a) and the 25 Hz output voltage waveform is shown in Fig. 7.2(c).

If another output frequency is required, say 16.5 Hz, the waveform will consist of three positive half cycles and three negative half cycles as shown in Fig. 7.2(d). Also, for 12.5 Hz output waveform it is required to trigger four positive half cycles and four negative half cycles. Therefore, if $m = \text{number of half supply cycle of frequency } f_i$, the output frequency $f_o$ will be

$$f_o = \frac{f_i}{m} \quad \text{where } m \text{ is integer } = 1,2,3,\ldots$$

(7.1)

Fig. 7.2 Single-phase to single-phase cycloconverter input and output waveforms.
From the output waveforms obtained in Fig. 7.2 (c) and (d), it is to be noted that the converter operates in the mode of half-cycle selection techniques with the triggering $\alpha = 0^\circ$ as discussed in Chapter Six. However, if it is required to control the value of the desired fundamental component of the output voltage waveform, the triggering angle $\alpha$ could be set to any value, e.g. $\alpha = 60^\circ$ as shown in Fig. 7.2(e), or make it variable throughout the whole output cycle using modulation techniques. The cycloconverter generating waveforms of Fig. 7.2(c) and (d) is called envelope cycloconverter, whereas cycloconverter generating waveforms of Fig. 7.2 (f) and (g) is called phase-angle controlled cycloconverter.

One distinct feature of the cycloconverters is that all of them are naturally commutated devices like controlled rectifiers. It is because of the necessity of producing natural commutation of the current between successive switching that the realizable output frequency is lower than the input frequency. Theoretically, the fundamental component of the highly distorted output voltage waveform is likely to be between $1/3$ and $2/3$ of the input frequency. Therefore, the cycloconverters are mostly step-down type.

It is possible to implement a cycloconverter in several different forms. The single-phase to single-phase version (Fig. 7.1) can also be constructed using two rectifier bridges connected back-to-back or inverse-parallel as shown in Fig. 7.3. By the controlled opening and closing of the switches of the two converters it is possible to fabricate output voltage waveforms having a fundamental component of the desired output frequency.

![Fig. 7.3 Single-phase to single-phase bridge type cycloconverter.](image)

The phase-angle controlled cycloconverter of Fig. 7.3 has exactly the same circuit topology as a dual converter. For a dual converter the firing angles of the converter switches are constant in time, as in bridge rectifier circuits, to result in rectifier operation with d.c. output. On the other hand, cycloconverter operation utilizes circuits in which the switch firing angles are functions of time and the output is a.c. voltage.
In the bridge type cycloconverter, each bridge is connected to the same a.c. source but no thyristor firing overlap can be permitted between them as this would short-circuit the supply. In an ideal dual converter the firing angles of the two converters are controlled so that their d.c. output voltages are exactly equal. If the positive bridge is gated at an angle $\alpha_p$ to produce a rectified output voltage $V_{dc}$, then the negative bridge is simultaneously gated at $\alpha_n$ to also produce the same output voltage $V_{dc}$, of consistent polarity. Thus the two bridges are delivering identical output d.c. voltages at any instant and the two firing angles must satisfy the relation,

$$\alpha_p - \alpha_n = \pi$$

(7.2)

The phase-angle controlled cycloconverter of Fig.7.3 can operates in two modes, *circulating current free mode* and *circulating current mode*:

1. **Circulating current free mode**

   Since the two bridges are acting alternately only one bridge carries load current at any given time. There is then no current circulating between the two bridges, and the mode of operation is usually called *circulating current free* operation or *noncirculating current* operation. The average value of the rectified load voltage was developed in Section 3.3 in Chapter Three as

   $$V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

   which can be re-written as

   $$V_{dc} = V_{do} \cos \alpha$$

   (7.3)

   where $V_{do} = \frac{2V_m}{\pi}$.

   Hence, when p-converter operating alone, the output voltage is positive half-cycles with average output voltage $V_{d1} = V_{do} \cos \alpha$. Also when n-converter operating alone, the output voltage is negative half-cycles with average output voltage $V_{d2} = -V_{do} \cos \alpha = -V_{d1}$. In order to maintain the circulating current free condition, the gating pulses to the thyristors must be controlled so that only the bridge carrying current is kept in conduction. The temporarily idle converter must be blocked by removing its gating pulses altogether.

2. **Circulating Current Mode**

   The circulating current mode is the standard mode of operation in which both the p-converter and n-converter are operating simultaneously.
The p-converter operates as rectifier and the n-converter operates in inversion mode. Thus the two bridges are delivering identical output d.c. voltages at any instant and the two firing angles must satisfy the relation as presented in Eq. (7.3).

An inter-group reactor is included, and allows current to circulate through the positive and negative converters. To keep current flow in both of the converters entirely continuous it is necessary to allow large amounts of circulating current. At times it can reach 57% of the output load current. The purpose of allowing for circulating current is to reduce the number of harmonics in the output voltage. This mode of operation also simplifies the optimal control strategy for the cycloconverter.

7.3 ANALYTICAL PROPERTIES OF THE SINGLE-PHASE TO SINGLE-PHASE CYCLOCONVERTER OUTPUT VOLTAGE WAVEFORMS

7.3.1 RMS Load Voltage
The \( V_{o(rms)} \) value of the output voltage waveform \( v_o(\omega t) \) in Fig. 7.2 is equal to the \( \text{rms} \) value of any of the half sinusoid sections. This is can be calculated as

\[
V_{o(rms)} = \sqrt{\frac{1}{\pi} \int_{0}^{\pi} V_m^2 \sin^2 \omega t \, d\omega t} = \sqrt{\frac{V_m^2}{\pi} \int_{0}^{\pi} \frac{1}{2} (1 - \cos 2\omega t) \, d\omega t}
\]

\[
= \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[ \pi - \alpha_p \right] + \frac{1}{2} \sin 2\alpha_p}
\]

(7.4)

It is to be noted from Eq. (7.4) that the \( \text{rms} \) value of the output voltage of a single-phase to single-phase cycloconverter is independent on the output frequency \( f_o \).

7.3.2 Fundamental Component of the Load Voltage Waveform
The Fourier coefficients \( a_1 \) and \( b_1 \) of the fundamental component of the output voltage waveform shown in Fig. 7.2 (\( f \)) can be calculated as:

If there are \( m \) half cycles of input in each half period of the output, then

\[
a_1 = \frac{2V_m}{m\pi} \left[ \int_{\alpha_p}^{\pi} \sin \omega_o t \cos \left( \frac{\omega_o t}{m} \right) d\omega_o t + \int_{\alpha_p}^{\pi} \sin \omega_o t \cos \left( \frac{\omega_o t + \pi}{m} \right) d\omega_o t \\
+ \int_{\alpha_p}^{\pi} \sin \omega_o t \cos \left( \frac{\omega_o t + (m-1)\pi}{m} \right) d\omega_o t \right]
\]
\[
\frac{V_m}{m\pi} \sum_{n=1}^{m} \left\{ \frac{m}{m+1} \left[ \cos \frac{n\pi}{m} + \cos \left( \alpha_p + \frac{\alpha_p + (n-1)\pi}{m} \right) \right]
+ \frac{m}{m-1} \left[ \cos \frac{n\pi}{m} + \sin \left( \alpha_p - \frac{\alpha_p + (n-1)\pi}{m} \right) \right] \right\}
\]

(7.5)

Similarly

\[
b_1 = \frac{2V_m}{m\pi} \left\{ \int_{\alpha_p}^{\pi} \sin \omega_o t \sin \left( \frac{\omega_o t}{m} \right) d\omega_o t + \int_{\alpha_p}^{\pi} \sin \omega_o t \sin \left( \frac{\omega_o t + \pi}{m} \right) d\omega_o t \right.
+ \int_{\alpha_p}^{\pi} \sin \omega_o t \sin \left( \frac{\omega_o t + (m-1)\pi}{m} \right) d\omega_o t \left. \right\}
\]

\[
= \frac{V_m}{m\pi} \sum_{n=1}^{m} \left\{ \frac{m}{m-1} \left[ \sin \frac{n\pi}{m} - \sin \left( \alpha_p - \frac{\alpha_p - (n-1)\pi}{m} \right) \right]
+ \frac{m}{m+1} \left[ \sin \frac{n\pi}{m} + \sin \left( \alpha_p - \frac{\alpha_p - (n-1)\pi}{m} \right) \right] \right\}
\]

(7.6)

The amplitude \( c_1 \) of the fundamental component is calculated as

\[
c_1 = \sqrt{a_1^2 + b_1^2}
\]

(7.7)

Higher harmonic components can be calculated by dividing the parameter \( m \) inside the square brackets of Eqs.(7.5) and (7.6 ) by the appropriate order of harmonic.

**Example 7.1**

The input voltage to the single-phase cycloconverter of Fig.7.1 is 240V, 50 Hz. The load resistance and inductance are 10 \( \Omega \) and 50 mH respectively. The frequency of the output voltage is 25 Hz. If the converter is to operate with \( \alpha_p = 75^\circ \),
(a) Sketch approximately to scale, the input supply voltages and the output voltage waveforms.
(b) Determine the \textit{rms} value of the output voltage.
(c) Determine the \textit{rms} value of the load current.
(d) Determine the input power factor.

\textbf{Solution}

(a) The input and output voltage waveforms are shown in Fig. 7.4.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig7.4}
\caption{Input and output voltage waveforms.}
\end{figure}

(b) From Eq. (7.4), the \textit{rms} value of the output voltage is

\begin{equation}
V_{o(rms)} = V_m \sqrt{\frac{1}{2\pi} \left[ \left( \pi - \alpha_p \right) + \frac{1}{2} \sin 2\alpha_p \right]}
\end{equation}

\begin{align*}
&= \sqrt{2} \times 240 \sqrt{\frac{1}{2\pi} \left[ \left( \pi - \frac{75^\circ \times \pi}{180^\circ} \right) + \frac{1}{2} \sin (2 \times 75^\circ) \right]} \\
&= 240 \sqrt{\frac{1}{\pi} \left[ (\pi - 0.4167\pi) + 0.25 \right]} = 195.4 \text{ V}
\end{align*}
The load impedance at 25 Hz is

\[ |Z| = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi \times 25 \times 0.050)^2} = 12.7 \, \Omega \]

\[ \theta = \tan^{-1} \frac{7.85}{10} = 38.13^\circ \]

The \textit{rms} value of the load current is

\[ I_o = \frac{V_o(rms)}{Z} = \frac{195.4}{12.7} = 15.38 \, \text{A} \]

(c) The input power factor:

\[ \text{RMS input current } I_s = I_o = 15.38 \, \text{A} \]

Input VA = \( I_s V_s = 15.38 \times 240 = 3692.6 \)

Output power \( P_o = I_o V_o(rms) \cos \theta = 15.38 \times 195.4 \cos 38.13^\circ \]

\[ = 2363.9 \, \text{W} \]

\[ PF = \frac{P_o}{VA} = \frac{2363.9}{3692.6} = 0.64 \]

7.4 MULTI-PHASE CYCLOCONVERTER

The single-phase cycloconverter circuits shown in Figs.7.1 and 7.3 are not practical circuits and are seldom used because of their nonsinusoidal output voltage. As many industrial applications require sinusoidal a.c. voltage and in order to construct a ‘clean’ sinusoidal output, multi-phase cycloconverters are much more effective in producing clean waveforms due to the variety of input to choose from. A nearly sinusoidal output voltage can be synthesized from three-phase input voltages by using three-phase controlled rectifiers as will be discussed in the following subsections.

7.4.1 Three-Phase to Single-Phase Cycloconverter

The three-phase to single-phase cycloconverter is shown in Fig.7.5 which is also called three-pulse cycloconverter since it uses three-phase half-wave converters.
The output voltage is positive when the thyristors of the p-converter are conducting, and the output voltage is negative when the thyristor of the n-converter are conducting. The positive (p) and negative (n) converters must operate alternatively without electrically overlap to prevent any circulating current when the two converters operate in the circulating current free mode. The average output voltage of the controlled rectifier of Fig. 7.5 is given in Eq.(3.20) which can be rewritten as,

\[ V_{dc} = v_o = \frac{3\sqrt{3}V_m}{2\pi} \cos \alpha \]  

(7.8)

With constant value of the triggering angle \( \alpha \) output voltage waveform of the three-phase to single-phase cycloconverter will be as depicted in Fig.7.6.
For phase-controlled cycloconverters with 50Hz supply the practical maximum output frequency is likely to be about 15Hz for single-way three-phase (three-pulse) rectifier and 30Hz for three-phase (six-pulse) bridge rectifiers. In the dual three-phase converter a.c. operation, the firing angles of the converter switches can be time varied, cycle by cycle. If the time varying firing angle is described as $\alpha(t)$, then the output voltage (no longer d.c.) has a peak amplitude $V_o$ given by

$$V_o = \frac{3\sqrt{3}V_m}{2\pi} \cos(\alpha(t)) \quad (7.9)$$

In Eq.(7.9), when the firing angle $\alpha(t)$ is varied with time in a periodic manner, this means that a process of discrete phase modulation is resulting. The supply frequency acts as a carrier signal, whereas the output frequency $\omega_o$ acts as the modulating frequency. This allows the output (modulated) voltage of the converter to be controlled in both amplitude and frequency independently.

A form of modulating function commonly used is

$$\alpha(t) = \pm \cos^{-1}(M\cos\omega_o t) \quad (7.10)$$

where $M$ is the modulation index which is the ratio of maximum modulating voltage to the maximum carrier voltage, and $\omega_o = 2\pi f_o$ ($f_o$ is the desired output frequency). For example, to convert a three-phase supply into a single-phase 15Hz output, it is necessary to use the modulating function (i.e., firing angle)

$$\alpha(t) = \pm \cos^{-1}(M\cos\omega_o t) = \pm \cos^{-1}(M\cos2\pi f_o t)$$

$$= \pm \cos^{-1}(M\cos15\pi t)$$

The output voltage waveform of the three-phase to single-phase cycloconverter with variable angle $\alpha$ is shown in Fig.7.7.

In Fig.7.7, it is clear that the cycloconverter output voltage waveform is made up of sections of sine waves which is called concurrent consecutive wave. This wave is highly distorted. The amount of distortion increases as the ratio output frequency/input frequency (i.e., modulating frequency / carrier frequency) increases. In addition to severe output voltage distortion the input current to a cycloconverter is usually significantly distorted. Also, the fundamental component of the input current lags the supply voltage, resulting in poor input power factor, irrespective of whether the load current is lagging, leading, or of unity power factor. Very often both input and output filters may be needed.
Fig. 7.7 Output voltage waveform of the three-phase to single-phase cycloconverter with variable value of the triggering angle $\alpha$.

The output voltage waveform shown in Fig. 7.7 can also be obtained from a three-phase half-wave rectifier shown in Fig. 7.8. The average of this controlled rectifier circuit varies as the cosine of the firing angle $\alpha$ (Equation 7.8). By successive variation of $\alpha$, a nearly sinusoidal output voltage can be obtained. Fabrication of such output voltage is illustrated in Fig. 7.9 as follows:

- At point ‘A’, $\alpha = 0^\circ$ and $V_{dc} = V_{do}$ (from Eq. (7.8)) where $V_{do} = 3\sqrt{3} \frac{V_m}{2\pi}$.
- At point ‘B’, $\alpha > 0$, hence $V_{dc} < V_{do}$ and so on for point ‘C’. The converter operates in rectifying mode.
- At point ‘D’, $\alpha = 90^\circ$, $V_{dc} = 0$ since $\cos 90^\circ = 0$ and the converter is idling.
- At points ‘E’, ‘F’, ‘G’, ‘H’ and ‘I’, $\alpha > 90^\circ$, $V_{dc} = \text{negative}$ and the converter operates in inversion mode.

The average value of the output voltage will be maximum at point ‘A’ ($\alpha = 0^\circ$) and zero at point ‘D’ ($\alpha = 90^\circ$). The firing angle is changed from $0^\circ$ to $90^\circ$ and then from $90^\circ$ to $180^\circ$ and back again to $90^\circ$ in appropriate steps.
Example 7.2

A three-phase to single-phase, three-pulse cycloconverter delivers power to a load rated at 100 V, 30 A, with power factor 0.8 lagging. Estimate the necessary input voltage and power factor.

Solution

From Eq.(7.9) the peak value of the required input voltage is given by

$$\sqrt{2} \times 100 = \frac{3\sqrt{3}V_m}{2\pi} \cos \alpha(t)$$

Solve for the worst case when $\cos \alpha(t) = 1$

$$V_m = \frac{2\pi \sqrt{2} \times 100}{3\sqrt{3}} = 171 \text{ V}$$

Hence the supply rms voltage per phase is
\[ V_s = \frac{171}{\sqrt{2}} = 120.93 \text{ V} \]

The \textit{rms} value of the specified single-phase load current is 30A. If this is shared equally between the three input phases, each input line (assuming sinusoidal operation) has an \textit{rms} current per phase,

\[ I_{\text{input}} = \frac{I_{\text{load}}}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 17.32 \text{ A} \]

The load power is

\[ P_{\text{load}} = V \cdot I \cdot PF = 100 \times 30 \times 0.8 = 2400 \text{ W} \]

The input power per-phase is therefore,

\[ P_{\text{input}} = \frac{P_{\text{load}}}{3} = \frac{2400}{3} = 800 \text{ W} \]

The input power factor is

\[ PF = \frac{P_{\text{input}}}{V_s I_{\text{input}}} = \frac{800}{120.93 \times 17.32} = \frac{800}{2094.5} = 0.382 \text{ lagging} \]

Note that this value of input power factor is the maximum possible value, with \( \alpha = 0 \). If \( \alpha \) is increased, the power factor will decrease.

\textbf{7.4.2 Three-Phase to Three-Phase Cycloconverter}

It is possible to implement a cycloconverter in several different forms. The three-phase to three-phase version of the cycloconverter with \( p = 3 \) supplying three-phase load is shown in Fig.7.10. In this figure, \( P_A \) is the positive converter of phase-A and \( N_A \) is the negative converter for phase-A. Similarly \( P_B \) and \( P_C \) are the positive converters of phase-B and phase-C respectively, whereas \( N_B \) and \( N_C \) are the negative converters of phase-B and phase-C respectively. The load voltage \( v_a \) is generated by \( P_A \) and \( N_A \) converters that are connected through an intergroup reactor. The firing schedules of thyristors in \( P_B - N_B \) converters and \( P_C - N_C \) converters are same as converters \( P_A - N_A \) but lag by 120˚ and 240˚ respectively. The output voltage waveforms of the three phases are shown in Fig.7.11 for clarity.

The average value of output voltage can be varied by varying the firing angles of the thyristors on conduction, whereas the frequency of the output voltage can be varied by changing the sequence of firing of the thyristors. Since each converter consists of three thyristors, thus in total

359
six thyristors per phase. This means, in all 18 thyristors are required for the whole circuit. This is why the cycloconverter considered as a complex device for a.c. to a.c. conversion, since it needs large number of thyristors with more complex triggering circuits.

![Diagram of three-phase to three-phase cycloconverter circuit.](image)

Fig. 7.10 Three-phase to three-phase cycloconverter circuit.

![Load voltage waveforms for three-phase to three-phase with p = 3 cycloconverter.](image)

Fig. 7.11 Load voltage waveforms for three-phase to three-phase with p = 3 cycloconverter.
Circulating current mode

The three-phase to three-phase cycloconverter can also be made to operate in circulating current mode to reduce the control problems arising with discontinuous load current. In this case an intergroup reactor may be connected between the p-converter and n-converter as shown in Fig.7.10 so that circulating current is allowed to circulate between them. In such scheme both the p-converter and n-converter are allowed to conduct simultaneously provided that the average output voltages of the two bridges are identical and must satisfy Eq.(7.2).

The intergroup reactor used in this scheme add cost to the circuit which is considered as disadvantage although it reduce the ripples and improve the quality of the output voltage waveform of the cycloconverter waveforms of this cycloconverter are depicted in Fig.7.12 for circulating current mode.

7.4.3 Three-Phase to Single-Phase Full-Wave (Six-Pulse) Cycloconverter

Fig.7.13 shows the schematic diagram of a three-phase to single-phase bridge type (p = 6) cycloconverter. Two three-phase, full-wave controlled bridge rectifier circuits of the form of P and N converters are connected back-to-back nature representing circulating current free operation. The positive group thyristors conduct for half the period of the output wave whereas the negative group thyristors conduct for the remaining half of the period.

A total of 12 thyristors are implemented in the circuit of this type of cycloconverter. However, the quality of the output voltage waveform is better than that of half-wave three-phase to single-phase converter circuit shown in Fig.7.8. For the three-phase, full-wave converter p = 6, and the peak load voltage may be inferred from Eq.(7.9) to be

$$V_o = \frac{3\sqrt{3}V_m}{\pi} \cos(\alpha(t)) \tag{7.11}$$

This is seen to be twice the value of the half-wave, three-pulse converter given in Eq.(7.9).

7.4.4 Three-Phase to Three-Phase Full-Wave (Six-Pulse) Cycloconverter

Fig.7.14 shows the schematic diagram of a three-phase to three-phase bridge type (p = 6) cycloconverter in circulating current mode. This converter is constructed using three three-phase to single-phase converters of Fig.7.13. A total of 36 thyristors are implemented in the circuit of this type of cycloconverter 12 for each phase. Fig.7.15 shows the same converter operating in free-circulating current mode.

The output voltages $v_o$ with six-pulse circulating current operation is given by Eq.(7.12) in reference [37] as
Fig. 7.12 Voltage waveforms for the three-phase to three-phase cycloconverter.
Fig. 7.13 Three-phase to single-phase full-wave (six-pulse) cycloconverter.

Fig. 7.14 Three-phase to three-phase full-wave (six-pulse) cycloconverter operating in circulating current mode.
Fig. 7.15 Three-phase to three-phase full-wave (six-pulse) cycloconverter operating in non-circulating current mode.
\[ v_o = k \frac{3\sqrt{3}V_m}{2\pi} \left[ M \sin \omega_o t + \sum_{p=1}^{\infty} \sum_{n=0}^{2n+1=6p+1} [A] [B] \right] \] (7.12)

where

\[ [A] = \left[ \frac{\alpha(6p-1)(2n+1)}{6p-1} + \frac{\alpha(6p+1)(2n+1)}{6p+1} \right] \]

\[ [B] = \sin[6p\omega_{in} t + (2n+1)\omega_o t] - \sin[6p\omega_{in} t - (2n+1)\omega_o t] \]

\[ k=1 \text{ for the six-pulse midpoint circuit} \]
\[ k=2 \text{ for the six-pulse bridge circuit} \]

From equation (7.12), it is seen that the output voltage contains large number of undesirable harmonic components.

### 7.5 ENVELOPE CYCLOCONVERTERS

The envelope cycloconverter is an alternative to the phase-controlled cycloconverter discussed in previous sections. The basic idea in operation of this earliest type of frequency changer is that the switches of Fig.7.1 in the envelope cycloconverter can operate continuously, like diodes. The firing angle ' \( \alpha \) ' of the component converter is not varied. It is kept constant normally at ' \( \alpha \) ' = 0° for the positive converter and ' \( \alpha \)' =180° for negative converter during the positive half cycle of the voltage whereas, during the negative half cycle, \( \alpha_p = 180^\circ \) and \( \alpha_n = 0^\circ \). In effect the rectifier valves, usually thyristors, act as on-off switches for the whole of a half cycle. This type of converter is generally called synchronous cycloconverter in which the commutation process of the conducting switches is entirely natural.

The output voltage waveform follows the profile of the a.c. supply voltage \( v_s \), as depicted in Fig.7.16 for simple single-phase version of the converter. These voltage waveforms shows frequency output of \( \frac{1}{2} f_i, \frac{1}{3} f_i \) and \( \frac{1}{4} f_i \) as shown in Fig.7.16 (b), (c) and (d) respectively. The major advantage of envelope converter is that it has a simple control ciruity. As such, the envelope cycloconverter is preferred if the desired output frequency is constant. However, in case the output voltage control is required, the input voltage has to be varied. The circulating currents may be avoided by turning off the conducting converter completely.
Fig. 7.16 Output voltage waveforms from envelope cycloconverter.

7.5.1 Performance Characteristics of a Single-Phase Envelope Cycloconverter

**Rms Load Voltage**

The *rms* value $V_{o(rms)}$ of the output voltage waveform $v_o(\omega t)$ in Fig. 7.16 (b) is equal to the *rms* value of any of the half sinusoid sections. This is can be calculated as

$$V_{o(rms)} = \sqrt{\frac{4}{4\pi} \int_0^{\pi} V_m^2 \sin^2\omega t \, d\omega} = \sqrt{\frac{V_m^2}{\pi} \int_0^{\pi} \frac{1}{2} (1 - \cos 2\omega t) \, d\omega}$$

$$= V_m \sqrt{\frac{1}{2\pi} \left[ (\pi - 0) + \frac{1}{2} (\sin 2\pi - \sin 0) \right]} = \frac{V_m}{\sqrt{2}} \quad (7.13)$$

It is to be noted from Eq.(7.13) that the *rms* value of the output voltage of a single-phase to single-phase envelope cycloconverter is independent on
the output frequency $f_o$ and has a value same as that of sinusoidal operation.

To generalize the rms value of the synchronous envelope cyclo-converter, if $m$ = number of half supply cycle of frequency $f_i$, the output frequency $f_o$ will be

$$f_o = \frac{2f_i}{m} \text{ where m is integer = 1,2,3, ...} \quad (7.14)$$

and the rms value $V_o(rms)$ of the output voltage waveform $v_o(\omega t)$ is

$$V_o(rms) = \frac{m}{\sqrt{m\pi}} \int_0^\pi V_m^2 \sin^2 \omega t \, d\omega t = \frac{V_m}{\sqrt{2}}$$

From Eq.(7.14), it is worth to note that the rms value of the output voltage waveform is the same for that of pure sinusoidal operation. However, this does not mean that the waveforms of Fig.7.16 (b), (c) and (d) are pure sinusoids and as a result the output voltage of this converter contains harmonics. These harmonics will be analysed in the following subsection.

**7.5.2 Harmonic Analysis of the Output Voltage Waveform**

**(A) Case when $T=2$ (Double half cycle)**

Let $T = 2$ is the number of conducting half cycles of the input supply voltage of frequency $f_i$ in a half period of the output voltage waveform of frequency $f_o$, hence the load voltage waveform $v_o$, for case of R-load shown in Fig.7.16(b), may be defined for a single-phase system as:

$$v_o = \sqrt{2} V_s \sin \omega t \quad 0 < \omega t < \pi$$

$$v_o = -\sqrt{2} V_s \sin \omega t \quad \pi < \omega t < 2\pi$$

$$v_o = -\sqrt{2} V_s \sin \omega t \quad 2\pi < \omega t < 3\pi$$

$$v_o = \sqrt{2} V_s \sin \omega t \quad 3\pi < \omega t < 4\pi$$

where $V_s = \text{rms value of the input supply voltage}$.

Fourier expansion of Eq.(7.15) gives the following results:

For $n \neq T$

$$a_o = 0 \quad (7.16)$$

$$a_n = \frac{\sqrt{2}V_s T}{\pi(T^2 - n^2)} \left[ 1 + 2\cos \frac{n\pi}{T} - 2\cos \frac{3n\pi}{T} - \cos \frac{4n\pi}{T} \right] \quad (7.17)$$
The amplitude \( c_n \) of the \( n \)th order harmonic component is calculated as

\[
c_n = \sqrt{a_n^2 + b_n^2}
\]  

(7.19)

and the phase angle \( \psi_n \) of the \( n \)th harmonic is

\[
\psi_n = \tan^{-1} \frac{a_n}{b_n}
\]  

(7.20)

For \( n = T \), the Fourier coefficients are:

\[ a_T = b_T = 0 \]  

(7.21)

\[ c_T \text{ and } \psi_n = 0 \]  

(7.22)

This means that the input frequency component is entirely suppressed and the fundamental component of the output voltage waveform is now defined as \( n = 1 \) in Eq. (7.15) with a value given by

\[
a_1 = \frac{\sqrt{2} V_s T}{\pi(T^2 - 1)} \left[ 1 + 2 \cos \frac{\pi}{T} - 2 \cos \frac{3\pi}{T} - \cos \frac{4\pi}{T} \right]
\]  

(7.23)

\[
b_1 = \frac{\sqrt{2} V_s T}{\pi(T^2 - 1)} \left[ 2 \sin \frac{\pi}{T} - 2 \sin \frac{3\pi}{T} - \sin \frac{4\pi}{T} \right]
\]  

(7.24)

For \( T = 2 \),

\[
a_1 = \frac{2\sqrt{2} V_s}{3\pi} \left[ 1 + 2 \cos \frac{\pi}{2} - 2 \cos \frac{3\pi}{2} - \cos 2\pi \right]
\]

\[
a_1 = \frac{2\sqrt{2} V_s}{3\pi} (1 + 0 - 0 - 1) = 0
\]

\[
b_1 = \frac{2\sqrt{2} V_s}{3\pi} \left[ 2 \sin \frac{\pi}{2} - 2 \sin \frac{3\pi}{2} - \sin 2\pi \right]
\]

\[
b_1 = \frac{2\sqrt{2} V_s}{3\pi} \left[ 2 + 2 - 0 \right] = \frac{8\sqrt{2} V_s}{3\pi} \rightarrow \therefore c_1 = b_1 = \frac{8\sqrt{2} V_s}{3\pi}
\]

The harmonic amplitude spectrum of the load voltage waveform for \( T = 2 \) and with \( R \)-load is shown in Fig. 7.17. It is clear that the output frequency of the waveform obtained from the envelope cycloconverter with \( T = 2 \) is half the input frequency (25 Hz for 50 Hz input). Higher order odd harmonics 3\(^{rd}\), 5\(^{th}\), 7\(^{th}\) and 11\(^{th}\) are only present in the spectrum. All even
harmonics (n=2,4,6...) are not present and the supply frequency component is entirely suppressed.

![Graph showing harmonic amplitude spectrum](image)

**Fig. 7.17** Harmonic amplitude spectrum of load voltage waveform for $T=2$ with $R$-load.

### 7.5.3 Three-Phase Cycloconverter with $T=2$ with $R$-Load

For medium and high power applications, three-phase cycloconverter is used. A three-phase version of a single-phase synchronous cycloconverter is shown in **Fig. 7.18** which consists of three single-phase cycloconverters $X,Y,$ and $Z$ feeding a three-phase star-connected resistive load in 4-wire configuration. The neutral wire is necessary to provide a return path to the load current for the three phases. Other type of connections such as 3-wire system or delta-connected load may not be feasible.

![Cycloconverter diagram](image)

**Fig. 7.18.** Three-phase synchronous cycloconverter.
For \( T=2 \), one can define the output voltage of Fig.7.16(b) in three-phase system as

\[
\begin{align*}
V_{Lj} &= \sqrt{2} V_s \sin(\omega t - \gamma_j) \quad 0 + \gamma_j < \omega t < \pi + \gamma_j \\
V_{Lj} &= \sqrt{2} V_s \sin(\omega t - \gamma_j) \quad \pi + \gamma_j < \omega t < 2\pi + \gamma_j \\
V_{Lj} &= -\sqrt{2} V_s (\sin \omega t - \gamma_j) \quad 2\pi + \gamma_j < \omega t < 3\pi + \gamma_j \\
V_{Lj} &= \sqrt{2} V_s \sin(\omega t - \gamma_j) \quad 3\pi + \gamma_j < \omega t < 4\pi + \gamma_j
\end{align*}
\]

where \( j = 1,2,3 \) and \( \gamma_1 = 0, \quad \gamma_2 = 2\pi / 3, \quad \gamma_3 = 4\pi / 3 \)

Fourier expansion of Eq.(7.25) \( n \neq T \) gives the following results:

\[
a_{o,j} = 0 \tag{7.26}
\]

\[
a_{n,j} = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \left[ \cos \frac{n\gamma_j}{T} + 2\cos \frac{n(\pi + \gamma_j)}{T} - 2 \cos \frac{n(3\pi + \gamma_j)}{T} - \cos \frac{n(4\pi + \gamma_j)}{T} \right] \tag{7.27}
\]

\[
b_{n,j} = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \left[ \sin \frac{n\gamma_j}{T} + 2\sin \frac{n(\pi + \gamma_j)}{T} - 2 \sin \frac{n(3\pi + \gamma_j)}{T} - \sin \frac{n(4\pi + \gamma_j)}{T} \right] \tag{7.28}
\]

For \( n = T \), the Fourier coefficients are:

\[
a_T = b_T = 0 \tag{7.29}
\]

\[
c_T \text{ and } \psi_n = 0 \tag{7.30}
\]

The harmonic amplitude spectrum for the three phases A, B, and C is found to be same as the harmonic spectrum for single phase presented in Fig.7.17. However, the phase-angle relationships of the three-phase voltage waveforms for \( R \)-load, for the case when \( T = 2 \), are shown in Fig. 7.19. It is clear that the fundamental component as well as the higher order harmonics are balanced in magnitudes but they are unbalanced in phase angles. With passive loads such as \( R \) and \( R-L \) loads there will be no problem in operation of the cycloconverter. However, with active load such as motors, the operation of the cycloconverter will encounter problems such as excessive heat, vibration and noise.
Fig. 7.19  Phase angle relationships for $T = 2$, R-load of the three phases A, B, and C.

7.5.4 Three-Phase Cycloconverter with $T=3$ with R-Load

In the following analysis three-phase version is considered for the case when $T=3$. Since the output waveforms for the three phase are same only they differ in phase lag of the three-phase system namely 0°, 120°, and 240°. The waveform of the output voltage when $T=3$ is shown in Fig. 7.16 (c) represent the case when $T=3$ with R-Load above and can be defined in general mathematical form as

\[
\begin{align*}
    v_{Lj} &= \sqrt{2} V_s \sin(\omega t - \gamma_j) & 0 + \gamma_j < \omega t < \pi + \gamma_j \\
    v_{Lj} &= -\sqrt{2} V_s \sin(\omega t - \gamma_j) & \pi + \gamma_j < \omega t < 2\pi + \gamma_j \\
    v_{Lj} &= \sqrt{2} V_s (\sin \omega t - \gamma_j) & 2\pi + \gamma_j < \omega t < 3\pi + \gamma_j \\
    v_{Lj} &= -\sqrt{2V_s} \sin(\omega t - \gamma_j) & 3\pi + \gamma_j < \omega t < 4\pi + \gamma_j \\
    v_{Lj} &= \sqrt{2V_s} \sin(\omega t - \gamma_j) & 4\pi + \gamma_j < \omega t < 5\pi + \gamma_j \\
    v_{Lj} &= -\sqrt{2V_s} \sin(\omega t - \gamma_j) & 5\pi + \gamma_j < \omega t < 6\pi + \gamma_j
\end{align*}
\]  

(7.31)

Fourier series analysis of Eq. (7.31) for the single-phase version of the converter gives
For $n \neq T$
\[ a_o = 0 \quad \text{(7.32)} \]
\[ a_n = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \quad \text{(7.33)} \]
\[ b_n = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \left[ 2 \sin \frac{n\pi}{T} + 2 \sin \frac{2n\pi}{T} - 2 \sin \frac{4n\pi}{T} - 2 \sin \frac{5n\pi}{T} - \sin \frac{6n\pi}{T} \right] \quad \text{(7.34)} \]

For $n = T$, the Fourier coefficients are:
\[ a_T = 0 \quad \text{(7.35)} \]
\[ b_T = \frac{\sqrt{2} V_s}{T} \quad \text{(7.36)} \]
\[ c_T = b_T = \frac{\sqrt{2} V_s}{T} \quad \text{(7.37)} \]

For $n = T$, the Fourier coefficients are:

Harmonic frequency spectrum of the output voltage waveform for one phase with $T = 3$ is shown in Fig.7.20. The fundamental frequency is $\frac{1}{3} T$ ($= \frac{1}{3} \times 50 = 16.66$ Hz). The $3^{rd}$ harmonic is zero, whereas the second significant harmonic is the $5^{th}$ one followed by the $7^{th}$. The other higher order harmonics are negligibly small.

Fig.7.20 Harmonic frequency spectrum of the output voltage waveform for one phase with $T=3$. 

\[ V_m = 220 \text{ V} \]
The Fourier analysis for the three-phase version gives:

For \( n \neq T \)

\[
a_{nj} = 0 \quad (7.38)
\]

\[
a_{nj} = \frac{\sqrt{2} V_s T}{\pi(T^2 - n^2)} \left[ \cos \frac{n\gamma_j}{T} + 2\cos \frac{n(\pi + \gamma_j)}{T} + 2 \cos \frac{n(3\pi + \gamma_j)}{T} 
- \cos \frac{n(4\pi + \gamma_j)}{T} - 2\cos \frac{n(5\pi + \gamma_j)}{T} 
- \cos \frac{n(6\pi + \gamma_j)}{T} \right] \quad (7.39)
\]

\[
b_{nj} = \frac{\sqrt{2} V_s T}{\pi(T^2 - n^2)} \left[ \sin \frac{n\gamma_j}{T} + 2\sin \frac{n(\pi + \gamma_j)}{T} + 2 \sin \frac{n(2\pi + \gamma_j)}{T} 
- 2 \sin \frac{n(4\pi + \gamma_j)}{T} 
- 2\sin \frac{n(5\pi + \gamma_j)}{T} - \sin \frac{n(6\pi + \gamma_j)}{T} \right] \quad (7.40)
\]

For \( n = T \), the Fourier coefficients are:

\[
a_{Tj} = \frac{\sqrt{2} V_s}{T} \sin (-\gamma_j) \quad (7.41)
\]

\[
b_{Tj} = \frac{\sqrt{2} V_s}{T} \cos (-\gamma_j) \quad (7.42)
\]

\[
c_{Tj} = \frac{\sqrt{2} V_s}{T} \quad (7.43)
\]

\[
\psi_{Tj} = \tan^{-1} \frac{a_{Tj}}{b_{Tj}} \quad (7.44)
\]

The phase-angle relationships of the load voltages for \( R \)-load, are shown in Fig. 7.21, for the case of \( T = 3 \). As in the case when \( T = 2 \), the harmonic amplitude spectrum for the three phases A, B, and C for \( T = 3 \) is found to be same as the harmonic spectrum for single phase presented in Fig.7.20. However, the phase-angle relationships of the three-phase voltage waveforms for \( R \)-load shown in Fig.7.20 for the fundamental component as well as the higher order harmonics are balanced in magnitudes but they are unbalanced in phase angles. Analysis of the output voltage waveform for any number of \( T = 4, 5, 6, 7, \ldots \) etc., reveals that the three-phase synchronous envelope cycloconverters generate output voltage waveforms that are balanced in magnitude but they are unbalanced in phase relationship.
Fig. 7.21 Phase angle relationships for $T = 3$, $R$-load of the three phases A, B and C.

**The Generalized Solution**

The generalisation of the solution of Fourier expansion of the output voltage waveform generated by synchronous envelop cycloconverter for any value of $T$ is derived in this section for single-phase and multi-phase systems. This generalized solution can be applied easily for any given waveforms as described hereinafter.

**1- Generalized equation for single-phase system**

Mathematical analysis of voltage waveforms for single-phase systems yields the following general Fourier analysis:

$$a_o = 0$$

(7.45)
\[ a_n = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \left[ 1 + 2 \sum_{k=1}^{k=T-1} \cos \frac{k n \pi}{T} - 2 \sum_{k=T+1}^{k=2T-1} \cos \frac{k n \pi}{T} \right.
\]
\[ - \cos \frac{2 T n \pi}{T} \left. \right] \quad (7.46) \]

\[ b_n = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \left[ 2 \sum_{k=1}^{k=T-1} \sin \frac{k n \pi}{T} - 2 \sum_{k=T+1}^{k=2T-1} \sin \frac{k n \pi}{T} \right.
\]
\[ - \sin \frac{2 T n \pi}{T} \left. \right] \quad (7.47) \]

where \( k = \text{integer } 1, 2, 3, \ldots \)

The \( n \)-th harmonic amplitude is given by:
\[ c_n = \sqrt{a_n^2 + b_n^2} \quad (7.48) \]

and the phase angle \( \psi_n \) of the \( n \)-th harmonic is
\[ \psi_n = \tan^{-1} \frac{a_n}{b_n} \quad (7.49) \]

2- Generalized equation for multi-phase system.

Mathematical analysis of voltage waveforms for multi-phase systems yields the following general Fourier analysis:

\[ a_{oj} = 0 \quad (7.50) \]

\[ a_n = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \left[ \cos n \gamma_j + 2 \sum_{k=1}^{k=T-1} \cos \frac{n(k \pi + \gamma_j)}{T} \right.
\]
\[ - 2 \sum_{k=T+1}^{k=2T-1} \cos \frac{n(k \pi + \gamma_j)}{T} - \cos \frac{n(2T \pi + \gamma_j)}{T} \left. \right] \quad (7.51) \]

\[ b_n = \frac{\sqrt{2} V_s T}{\pi (T^2 - n^2)} \left[ \sin n \gamma_j + 2 \sum_{k=1}^{k=T-1} \sin \frac{n(k \pi + \gamma_j)}{T} \right.
\]
\[ - 2 \sum_{k=T+1}^{k=2T-1} \sin \frac{n(k \pi + \gamma_j)}{T} - \sin \frac{n(2T \pi + \gamma_j)}{T} \left. \right] \quad (7.52) \]
The \( n \)th harmonic amplitude is given by:

\[
\epsilon_{nj} = \sqrt{a_{nj}^2 + b_{nj}^2}
\]  
(7.53)

and the phase angle \( \psi_n \) is:

\[
\psi_{nj} = \tan^{-1} \frac{a_n}{b_n}
\]  
(7.54)

Application of the generalized equations from Eqs.(7.50) to (7.54) to both single-phase and multi-phase system give the same results obtained from the individual analysis of each waveform. From the previous results, it is found that when \( T \) is even integer number (\( T = 2,4,6,8,\ldots \)) the Fourier coefficient of the supply frequency components \( a_{Tj} \) and \( b_{Tj} \) are always equal to zero, while for odd values of \( T =3,5,7,9,\ldots \), the supply frequency components \( a_{Tj} \) and \( b_{Tj} \) have significant values. Fig.7.22 shows the frequency spectrum for \( T = 4 \). The phase relationship of the three phases is also shown in Fig.7.23 which shows unbalanced phase-angle between the three individual phases A,B, and C.

The problem of unbalanced phase-angle relationships, can be solved using a multiple of 2\( \pi \) phase shifting technique, which is based on shifting the load voltage \( v_{Lj} \) of phase B or phase C or both of them, by multiple of 2\( \pi \), with phase A taken as a reference, i.e when \( \gamma_A = 0 \), the other phase angles will be:

\[
\gamma_B = (2 \pi/3) + 2\pi \\
\gamma_C = (4 \pi/3) + 4\pi
\]

It is found that this technique makes the phase displacement angles of the \( n \)th harmonics balanced except when \( n \) is multiple of 3, in this case the phase displacement angles will be in-phase for all cases. The result of implementing this technique is shown in Fig.7.24 as an example for \( T=4 \).

![Fig.7.22 Harmonic frequency spectrum of the output voltage waveform for one phase with \( T=4 \).](image)
Fig. 7.23 Harmonic amplitude spectrum (of phase A, B, and C) and phase angle relationships for $T = 4$, R-load before phase-angle corrections.

Fig. 7.24 Phase-angle relationships of phase A, B, and C for $T = 4$, R-load after phase-angle correction.
7.6  HARMONICS REDUCTION IN ENVELOPE CYCLOCONVERTER

As it is seen from the previous analysis that the output voltage waveform of the envelope cycloconverter contains harmonics. To reduce these harmonics it is necessary to generate wave shape which is nearly sinusoidal or stepped wave. For single phase source, to generate stepped wave, it is necessary to use some form of input transformer connection. Conduction in successive phases of the transformer secondary windings is made in logic sequence, with respect to the supply voltages, so as to provide an output voltage envelope that is approximately a stepped waveform of the desired output frequency.

In the circuit of Fig. 7.25 (a), where Triacs are adopted, the voltage applied across the load can be the full supply voltage of either polarity or some fraction $V_{m1}$, depending on the transformer secondary tap setting. The switches are shown as triacs but can equally well be pairs of inverse-parallel connected silicon controlled rectifiers or any other bidirectional switch. With an ideal supply and ideal switches waveform of the shape shown in Fig. 7.25(b) can be obtained.

![Circuit diagram of envelope cycloconverter](image)

![Load voltage waveforms](image)

Fig. 7.25  Single-phase envelope cycloconverter: (a) Circuit diagram for two-pulse operation and (b) Load voltage waveforms with output frequency $f_o = (1/3) f_{in}$.  

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Basically, a single-phase envelope cycloconverter can be considered to be composed of two converters connected back to back as shown in Fig.7.26. The types of voltage waveforms that can be obtained using envelope converters shown in figure can be improved by increasing the number of pulses using three-phase or multi-phase systems. The waveform is obtained by properly selecting the input voltages to the converter. As with rectifier circuit operation an increase of pulse number reduces the voltage ripple and the higher harmonic content.

![Fig. 7.26](image)

**Fig. 7.26** Three-phase to single-phase envelope cycloconverter: (a) Circuit diagram for two-pulse operation and (b) Load voltage waveforms with output frequency \( f_o = \frac{1}{2} f_{in} \).

### 7.7 THE MATRIX CONVERTER

A matrix converter is defined as an ac to ac converter with a single stage of conversion. It utilizes bidirectional controlled switch to achieve automatic conversion of power. In its topology, the matrix converter can be considered as a three-phase to three-phase forced commutated cycloconverter which, in its simplest form, consists of nine bi-directional switches that allow any output phase to be connected to any input phase as it is shown in Fig.7.27. The input terminal of the converter are connected to a voltage source, usually the grid, while the output terminal are connected to a current source like an induction motor. Only one of the three switches connected to the same output phase can conducts at any instant of time.
The matrix converter has several advantages over other type of ac to ac converters in that

- The output voltage waveforms generated by this converter has minimal higher order harmonic and no subharmonics.
- The bidirectional switches make it possible to have a controllable power factor input since this converter has inherent bidirectional energy flow capability (improved power factor and regeneration capability).
- The lack of d.c. links ensures it has a compact design.
- Generation of output voltage with the desired amplitude and frequency.
- Sinusoidal input current and sinusoidal output voltage.

The main disadvantage of the matrix converter is that it requires more semiconductor devices than a conventional ac to ac indirect power frequency changer.

![Three-phase to three-phase matrix converter](image)

Fig. 7.27 Schematic circuit of a three-phase to three-phase matrix converter.

In the matrix converter, if it is assumed that all the switches are ideal and no energy storage components are present between the input and output side of the converter, then, the instantaneous input power must always be equal to the instantaneous output power. Due to this power invariance feature, the phase angles between the voltages and currents at the input can be controlled to give unity displacement factor for any loads. Therefore, for the converter, it is necessary to use ideal bidirectional switches, capable of conducting current and blocking voltage for both polarities depending on the actual control signal.

The required output voltage waveforms are generated directly from the input voltage waveforms and each output voltage waveform is synthesized by sequential sampling of the input voltage waveforms. The sampling rate has to be set much higher than input and output frequencies,
the number of input and output phases do not have to be equal so that rectification, inversion, and frequency conversion are all realizable. The output voltage waveforms are therefore composed of segments of the input voltage waves. The lengths of each segment are determined mathematically to ensure that the average value of the actual output waveform within each sampling period tracks the required output waveform.

**Methods of matrix converter control**

There are three methods of matrix converter control,

- Venturini-analysis of function transfer
- Pulse width modulation
- Space vector modulation

The Venturini principle can be explained initially using a three-phase output. Consider a three-phase output using a three-phase input voltage as depicted in Fig.7.27, switching elements $S_{Aa} - S_{Cc}$ are nine bidirectional switches connecting the input phases (a, b, and c) to any output phase (A, B, and C) at any instant. Only one of the three switches is turned on at any given time, and this ensures that the input of a matrix converter, which is a voltage source, is not short-circuited while a continuous current is supplied to the load.

During phase connection process, the three voltages $v_{AN}, v_{BN}$ and $v_{CN}$ are related to the three input voltages by the following mathematical formula:

\[
\begin{pmatrix}
  V_{AN} \\
  V_{BN} \\
  V_{CN}
\end{pmatrix} =
\begin{pmatrix}
  S_{Aa} & S_{Ba} & S_{Ca} \\
  S_{Ab} & S_{Bb} & S_{Cb} \\
  S_{Ac} & S_{Bc} & S_{Cc}
\end{pmatrix}
\begin{pmatrix}
  V_{an} \\
  V_{bn} \\
  V_{cn}
\end{pmatrix}
\]  

(7.55)

or

\[ V = S \cdot v \]

where

\[
V =
\begin{pmatrix}
  V_{AN} \\
  V_{BN} \\
  V_{CN}
\end{pmatrix}, \quad
S =
\begin{pmatrix}
  S_{Aa} & S_{Ba} & S_{Ca} \\
  S_{Ab} & S_{Bb} & S_{Cb} \\
  S_{Ac} & S_{Bc} & S_{Cc}
\end{pmatrix}
\quad \text{and} \quad
v =
\begin{pmatrix}
  V_{an} \\
  V_{bn} \\
  V_{cn}
\end{pmatrix}
\]

S = connection matrix consists of the switching variables $S_{Aa}$ through $S_{Cc}$.

For star-connected load as shown in Fig.7.27, the output phase current are related to the input phase current by
Theoretically, with nine bi-directional switches the matrix converter can have \(2^9 = 512\) different switching states combinations. But not all of them can be usefully employed. Because the converter is supplied by a voltage source and usually feeds an inductive load, the input phases must not be short-circuited and the output currents should not be interrupted. These rules imply that one and only one bi-directional switch per output phase must be switched on at any instant. By this constraint, in a three-phase to three-phase matrix converter only 27 switching combinations are permitted in practice.

Although the matrix converter has the features that have been briefly described above it might be surprising to establish that this converter topology, today, has not found a wide utilization yet. The reasons for that are due to the number of practical implementation problems that have slowed down the development of this type of ac-to-ac converter.

**PROBLEMS**

7.1 For the output voltage waveform \(v_o(\omega t)\) of Fig. 7.4 with peak voltage 200V, calculate the values of the rms voltage \(V_o\) for the triggering angle \(\alpha\): (a) 30\(^\circ\), (b) 45\(^\circ\), (c) 90\(^\circ\), (d) 125\(^\circ\), and (e) 150\(^\circ\).

[Ans: (a) 139.37 V, (b) 131.36 V, (c) 114.84 V, (d) 104.38 V, (e) 70 V]

7.2 Show that the fundamental component of waveform \(v_o(\omega t)\) of Fig. 7.4 is zero. What is the frequency of the lowest order harmonic?

7.3 In the single-phase cycloconverter shown in Fig. 7.3, the input supply voltage is 240V, 50Hz. The load is a pure resistance and the output frequency is 12.5Hz. It is required to:

(a) Sketch the waveforms for the input voltage versus output voltage \(v_o\) for \(\alpha = 0^\circ\) and \(\alpha = 90^\circ\).

(b) Calculate the rms value of the output voltage at \(\alpha = 0^\circ\) and \(\alpha = 90^\circ\).

[Ans: (b) 240 V, 169.73 V]
7.4 A three-phase, four-wire, half-wave rectifier dual converter operated from a 400V, 50-Hz three-phase supply. Calculate the average value of the load voltage for firing angles (a) 30°, (b) 45°, (c) 90°, (d) 125°, and (e) 150°.

[Ans: (a) 405.26 V, (b) 330 V, (c) 0 V, (d) -268.37 V, (e) -405.26 V]

7.5 The input voltage to the single-phase to single-phase cycloconverter is 240V(rms), 50Hz. The load resistance is 10Ω and the load inductance is \( L = 50 \text{ mH} \). The frequency of the output voltage is 25Hz. If the converter is operating with a delay angle \( \alpha_p = 2\pi/3 \),

(a) Sketch the circuit configuration of the converter.
(b) Sketch the output voltage waveform.
(c) Determine the \( \text{rms} \) value of the output voltage.
(d) Determine the impedance of the load at the output frequency.
(e) Calculate the \( \text{rms} \) value of the load current.
(f) Determine the input power factor of the converter.

[ Ans: (c) 106 V, (d) 12.7 \( \Omega \), (e) 8.34 A, (f) 0.371 (lagging)]

7.6 A single-phase cycloconverter operating in circulating current free dual converter with input 220 V, 50 Hz is used to produce an output voltage waveform of the general form (Fig.7.5). The output frequency is one-third of the input frequency, and \( \alpha = 45^\circ \). Calculate the amplitude of the fundamental voltage.

[Ans: 135.85 V]

7.7 A three-phase to single-phase three-pulse cycloconverter delivers an output of 240 V, 70 A to a load of 0.9 power factor lagging. Assume that the switches of the converter are ideal and that the output current is nearly sinusoidal. Calculate the input voltage and input power factor.

[Ans: \( V_s = 290 \text{ V}, PF = 0.4315 \) ]

7.8 A three-phase to three-phase, three-pulse cycloconverter operating in circulating current-free such that the input / output frequency ratio is \( f_i/f_o = 9 \). What are typical frequencies of the low-order higher harmonics?

[Ans: \((2/9)f_o, (1/3)f_o, (4/9)f_o, (5/9)f_o, (2/3)f_o, (7/9)f_o, \ldots \) ]