Chapter 23

1

ELECTRIC FIELDS

1

Lecture 05

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Electric Field of a Continuous Charge Distribution

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Electric Field of a Continuous Charge Distribution

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The distances between charges in a group of charges may be much smaller than the distance between the group and a point of interest.

In this situation, the system of charges can be modeled as continuous.

The system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

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Electric Field – Continuous Charge Distribution

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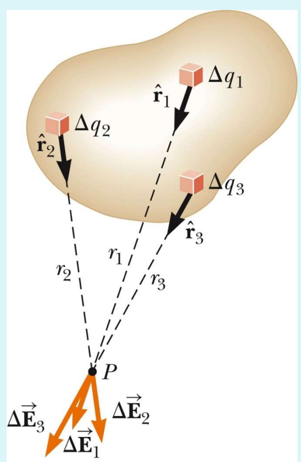
- Procedure:
  - Divide the charge distribution into small elements, each of which contains Δq.
  - Calculate the electric field due to one of these elements at point P.
  - Evaluate the total field by summing the contributions of all the charge elements.



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Electric Field – Continuous Charge Distribution, equations

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- For the individual charge elements

$$\Delta\vec{E} = k_e \frac{\Delta q}{r^2}\hat{r}$$

- Because the charge distribution is continuous

$$\vec{E} = k_e \sum_i \frac{\Delta q_i}{r_i^2}\hat{r}$$

- When $\Delta q \to 0$:

$$\vec{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{r}$$

$$\vec{E} = k_e \int \frac{dq}{r^2}\hat{r}$$

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Charge Densities

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Volume charge density: when a charge is distributed evenly throughout a volume

$$\rho = \frac{\text{Charge}}{\text{Volume}} = \frac{Q}{V} \text{ with units } \frac{C}{m^3}$$

Surface charge density: when a charge is distributed evenly over a surface area

$$\sigma = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{A} \text{ with units } \frac{C}{m^2}$$

Linear charge density: when a charge is distributed along a line

$$\lambda = \frac{\text{Charge}}{\text{Length}} = \frac{Q}{\ell} \text{ with units } \frac{C}{m}$$

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Amount of Charge in a Small Volume

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If the charge is **uniformly** distributed over a volume, surface, or line, the amount of charge, dq, is given by

For the volume:

$$dq = \rho dV$$

For the surface:

$$dq = \sigma dA$$

For the length element:

$$dq = \lambda d\ell$$

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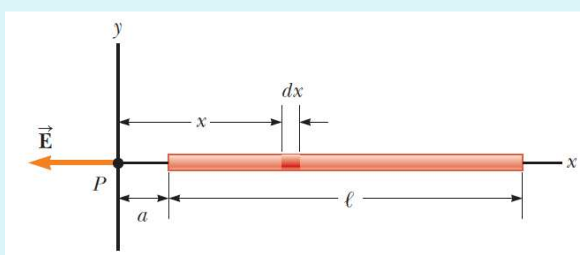
Example 1:

The Electric Field Due to a Charged Rod

A rod of length (ℓ) has a uniform positive charge per unit length (λ) and a total charge (Q). Calculate the electric field at a point (P) that is located along the long axis of the rod and a distance (a) from one end.



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Example 1: The Electric Field Due to a Charged Rod

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$$dq = \lambda dx \quad , r = x$$

$$E = k_e \int \frac{dq}{r^2}$$

$$E = k_e \int_a^{\ell+a} \frac{\lambda dx}{x^2}$$

$$E = k_e\lambda \int_a^{\ell+a} \frac{dx}{x^2} = k_e\lambda \left(\frac{-1}{x}\right)\Big|_a^{a+\ell}$$

$$E = k_e\lambda \left(\frac{-1}{a+\ell} - \frac{-1}{a}\right)$$

$$E = k_e\lambda \left(\frac{1}{a} - \frac{1}{a+\ell}\right)$$

$$E = k_e\lambda \left(\frac{a+\ell-a}{a(a+\ell)}\right)$$

$$E = \frac{k_e\lambda\ell}{a(\ell+a)}$$

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Example 1: The Electric Field Due to a Charged Rod

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$$E = k_e\lambda \int_a^{\ell+a} \frac{dx}{x^2} = \frac{k_e\lambda\ell}{a(\ell+a)} = \frac{k_eQ}{a(\ell+a)}$$

- λ : charge per unit length (C/m)
- ℓ : Length (m)
- Q: total charge (C)
- a: distance from (P) to nearest end (m)
- **If $a \gg \ell$:**

$$E = \frac{kQ}{a^2}$$

- That is exactly the form for a point charge.

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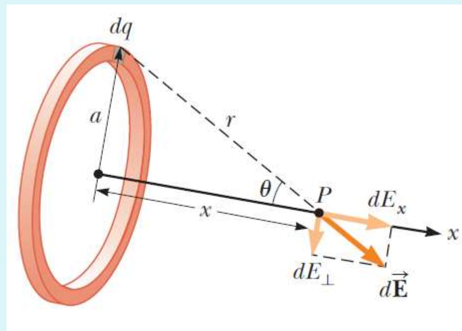
Example 2:
The Electric Field of a Uniform Ring of Charge

A ring of radius (a) carries a uniformly distributed positive total charge (Q). Calculate the electric field due to the ring at a point (P) lying a distance (x) from its centre along the central axis perpendicular to the plane of the ring.



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Example 2: The Electric Field of a Uniform Ring of Charge

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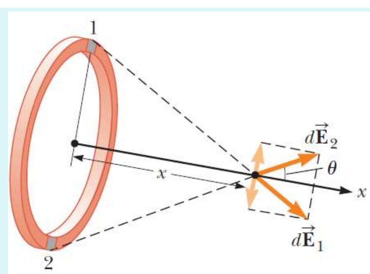
- The electric field has two components:
- By symmetry, the **perpendicular component** is:

$$E_{\perp} = 0$$

- The **x-component** is:

$$E_x = \frac{k_e Q x}{(x^2 + a^2)^{3/2}}$$

- a: radius of the ring (m)
- Q: total charge (C)
- x: the distance from the centre of the ring to point (P) laying along the central axis perpendicular to its plane.



for locations **far away** from the ring: $(x \gg a)$:

$$(x^2 + a^2)^{3/2} \cong (x^2)^{3/2} = x^3$$

Then:

$$E_x = \frac{k_e Q}{x^2}$$

the ring acts like a **point charge**

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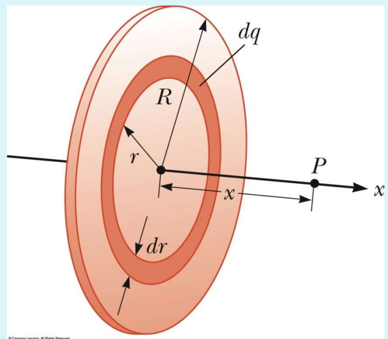
Example 3: The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density (σ) . Calculate the electric field at a point (P) that lies along the central perpendicular axis of the disk and a distance x from the center of the disk.



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Example 3: The Electric Field of a Uniformly Charged Disk

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- The electric field has two components:
- By symmetry, the **perpendicular component** is:

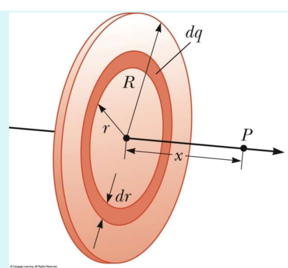
$$E_{\perp} = 0$$

- The **x-component** is:

$$E_x = 2\pi k_e \sigma \left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)$$

- OR:

$$E_x = \frac{\sigma}{2\epsilon_\circ}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)$$



R: radius of the disk (m)
σ: surface charge density (C/m^2)
x: the distance from the centre of the disk to point (P) laying along the central axis perpendicular to its plane.

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Example 3: The Electric Field of a Uniformly Charged Disk

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for locations far away from the disk: $(x \gg R)$. Then:

$$E_x = \frac{k_e Q}{x^2}$$

the ring acts like a point charge.

for locations very close to the disk: $(x \ll R)\ or\ (R \to \infty)$. Then:

$$\left(1 - \frac{x}{(x^2 + R^2)^{3/2}}\right) \cong \left(1 - \frac{x}{R^3}\right) \cong 1$$

Then:

$$E_x = \frac{\sigma}{2\epsilon_\circ}$$

Which shows that the electric field due to an infinite plane of charge is uniform throughout space.

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Charged rod 1

Friday, 29 January, 2021 19:59

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

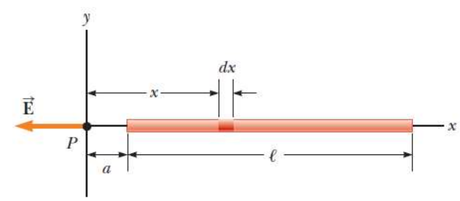
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A rod $L = 14cm$ long is uniformly charged and has a total charge of $Q = -22\ \mu C$. Determine the magnitude of the electric field along the axis of the rod at a point $36\ cm$ from its center.

Answer:	
$E = k_e\lambda \int_a^{L+a} \frac{dx}{x^2}$	$E = \frac{k_e \lambda L}{a(L+a)} = \frac{k_e Q}{a(L+a)}$
When $a \gg L$:	$E \cong \frac{k_e Q}{a^2}$



$L = 14cm = 0.14m$

$a = 0.36 - 0.07 = 0.29m$

$Q = -22\mu C = -22 \times 10^{-6} C$

$$E = \frac{9 \times 10^9 \times 22 \times 10^{-6}}{0.29 \times (0.14 + 0.29)} = 1.6 \times 10^6\ N/C$$

$$\vec{E} = 1.6 \times 10^6\ N/C, \hat{\imath}$$

Let's assume the rod is lying along the x axis, dx is the length of one small segment, and dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is:

$$\lambda = \frac{Q}{L} = \frac{dq}{dx}$$

$$dq = \lambda\, \mathrm{dx}$$

Find the magnitude of the electric field at P due to one segment of the rod having a charge dq:

$$dE = \frac{k\, dq}{x^2} = \frac{k\lambda\, dx}{x^2}$$

Find the total field at P:

$$E = k\lambda \int_a^{L+a} \frac{dx}{x^2}$$

Noting that k and $\lambda = \frac{Q}{L}$, are constants and can be removed from the integral, evaluate the integral:

$$E = k\lambda \left(\frac{-1}{x} \right) \Bigg|_{a}^{L+a}$$

$$E = k\lambda \left(\frac{-1}{L+a} + \frac{1}{a} \right)$$

$$E = \frac{k_e \lambda L}{a(L+a)} = \frac{k_e Q}{a(L+a)}$$

IF P is far from the rod ($a \gg L$), then L, in the denominator can be neglected and:

$$a(L+a) \cong a^2$$

$$E = \frac{kQ}{a^2}$$

That is exactly the form you would expect for a point charge.

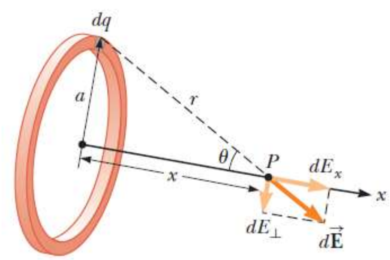
On the other hand, if $a \rightarrow 0$, which corresponds to sliding the bar to the left until its left end is at the origin, then $E \rightarrow \infty$.

★ Charged ring

Friday, 29 January, 2021 19:59

A uniformly charged ring of radius $10\ cm$ has a total charge of $75\ \mu C$. Find the electric field on the axis of the ring at (a) $1\ cm$, (b) $5\ cm$, (c) $30\ cm$ and (d) $1000\ cm$ from the center of the ring.

Answer:	
By symmetry	$E_\perp = 0$
	$E_x = \dfrac{k_e\, Q\, x}{(a^2 + x^2)^{3/2}}$
When $x \gg a$	$E_x \cong \dfrac{k_e Q}{x^2}$
When $x = 0$	$E_x = 0$
When $x \ll a$	$E_x \cong \dfrac{k_e Q}{a^3} x$



$a = 10cm = 0.1m$

$Q = 75\mu C = 75 \times 10^{-6} C$

$x = 1000cm = 10m$

$$E = \frac{9 \times 10^9 \times 75 \times 10^{-6} \times 10}{(0 \cdot 1^2 + 10^2)^{3/2}} = 6.748 \times 10^3\ N/C$$

$x \gg a$

$$E = \frac{9 \times 10^9 \times 75 \times 10^{-6}}{10^2} = 6.75 \times 10^3\ N/C$$

The figure shows the electric field contribution $d\vec{E}$ at P due to a single segment of charge dq at the top of the ring.

$$dE = \frac{k\, dq}{r^2} = \frac{k\, dq}{x^2 + a^2}$$

This field vector can be resolved into components dE_x parallel to the axis of the ring and $dE_\perp$ perpendicular to the axis.

Because of the symmetry of the situation, the perpendicular components of the field cancel. That is true for all pairs of segments around the ring, so:

$$E_\perp = \int dE_y = 0$$

Now, we can ignore the perpendicular component of the field and focus solely on the parallel components, which simply add.

Evaluate the parallel component of an electric field contribution from a segment of charge dq on the ring:

$$E_x = \int dE_x = \int dE \cos\theta$$

From the geometry:

$$\cos\theta = \frac{x}{\sqrt{x^2 + a^2}}$$

$$E_x = \int \frac{k\,dq}{x^2 + a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Integrate over the circumference of the ring to obtain the total field at P :

$$E_x = \int \frac{kx}{(x^2 + a^2)^{3/2}} dq$$

$$E_x = \frac{kx}{(x^2 + a^2)^{3/2}} \int_0^Q dq$$

$$E_x = \frac{kxQ}{(x^2 + a^2)^{3/2}}$$

This result shows that the field is zero at $x = 0$:

At $x = 0,\ \ E = 0$

Furthermore, If $x >> a$, then:

$$(x^2 + a^2)^{3/2} \cong (x^2)^{3/2} \cong x^3$$

$$E_x = \frac{kQ}{x^2}$$

So the ring acts like a point charge for locations far away from the ring.

Suppose a negative charge is placed at the center of the ring, and displaced slightly by a distance $x << a$ along the x axis. When the charge is released:

$$(x^2 + a^2)^{3/2} \cong (a^2)^{3/2} \cong a^3$$

$$E_x = \frac{kQ}{a^3} x$$

Therefore, the force on a charge $-q$ placed near the center of the ring is:

$$F = -\frac{kqQ}{a^3}x$$

Because this force has the form of Hooke's law, the motion of the negative charge is described with the particle in simple harmonic motion!

★ Charged disk

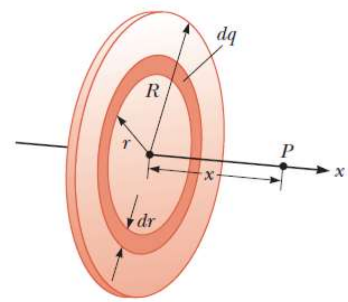
Friday, 29 January, 2021 20:00

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A uniformly charged disk of radius $35.0\ cm$ carries charge with a density of $7.9 \times 10^{-3}\ C/m^2$. Calculate the electric field on the axis of the disk at (a) $5\ cm$, (b) $10\ cm$, (c) $50\ cm$ and (d) $200\ cm$ from the center of the disk.

Answer:		
By symmetry:	$E_\perp = 0$	
	$E_x = 2k_e\pi\sigma x \int_0^R \frac{r\,dr}{(r^2+x^2)^{3/2}}$	$E_x = 2k_e\pi\sigma\left[1 - \frac{x}{(x^2+R^2)^{1/2}}\right]$
When $x \gg R$:	$E_x \cong \frac{k_e Q}{x^2}$	
When $x \ll R$:	$E_x \cong 2k_e\pi\sigma \cong \frac{\sigma}{2\epsilon_\circ}$	



Solution:

$R = 0.35m$

$\sigma = 7\cdot 9 \times 10^{-3}\ C/m^2$

$x = 0.05m$

$$E = 2 \times 9 \times 10^9 \times \pi \times 7.9 \times 10^{-3} \times \left(1 - \frac{0.05}{(0.05^2 + 0.35^2)^{1/2}}\right)$$

$$= 3.836 \times 10^8\ N/C$$

Because the disk is continuous, we are evaluating the field due to a continuous charge distribution rather than a group of individual charges.

$$dE = \frac{k\,\mathrm{d}q}{r^2}$$

If the disk is considered to be a set of concentric rings, we can use the equation for the field created by a single ring of radius a, and sum the contributions of all rings making up the disk. By symmetry, the field at an axial point must be along the central axis.

Find the amount of charge dq on the surface area of a ring of radius r and width dr as shown:

$$\sigma = \frac{Q}{A} = \frac{\mathrm{d}q}{\mathrm{d}A} = \frac{\mathrm{d}q}{2\pi r\,\mathrm{d}r}$$

$$dq = 2\pi r\sigma\, dr$$

Use the equation for the field created by a single ring (with a replaced by r and Q replaced by dq) to find the field due to the ring:

$$dE_x = \frac{kx}{(x^2 + r^2)^{3/2}}(2\pi r\sigma\, dr)$$

To obtain the total field at P, integrate this expression over the limits $r = 0$ to $r = R$, noting that x is a

constant in this situation:

$$E_x = 2k\pi\sigma x \int_0^R \frac{r}{(x^2 + r^2)^{3/2}} dr$$

Using: $\int \frac{x\, dx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$

$$E_x = 2k\pi\sigma\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

$$E_x = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

This result is valid for all values of $x > 0$. For large values of x, the result above can be evaluated by a series expansion (Taylor expansion) and shown to be equivalent to the electric field of a point charge Q.

$(1 + a)^n = 1 + na\, , a = \frac{R^2}{x^2}\, , n = \frac{1}{2}$

$$1 - \frac{x}{\sqrt{R^2 + x^2}} = 1 - \frac{x}{x\sqrt{1 + \frac{R^2}{x^2}}} = 1 - \left(1 + \frac{R^2}{x^2}\right)^{-1/2} = 1 - \left(1 - \frac{1}{2}\frac{R^2}{x^2}\right) = \frac{R^2}{2x^2}$$

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

$$E_x = 2k\pi\sigma\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

$$E_x = 2k\pi\frac{Q}{\pi R^2}\frac{R^2}{2x^2} = \frac{kQ}{x^2}$$

We can calculate the field close to the disk along the axis by assuming $x \ll R$, or if the disk becomes an infinite plane of charge $(R \to \infty)$; in this case, the expression in brackets reduces to unity to give us:

$$E_x = 2k\pi\sigma = \frac{\sigma}{2\varepsilon_0}$$

This result is independent of the position at which you measure the electric field. Therefore, the electric field due to an infinite plane of charge is uniform throughout space.

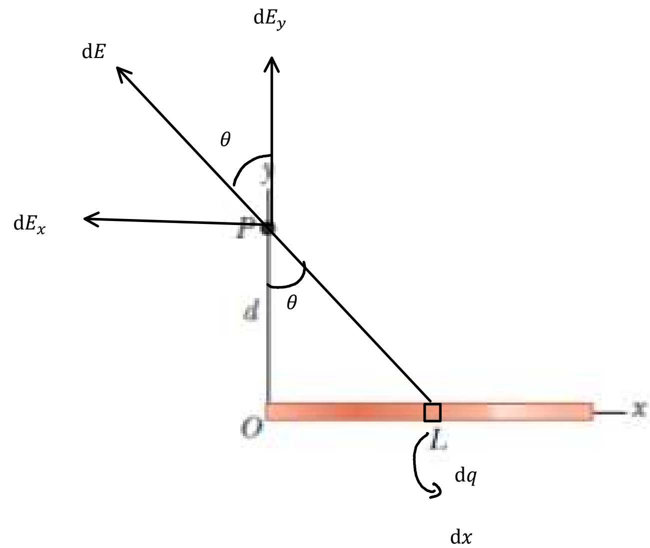
★ Charged rod 2

Friday, 29 January, 2021 20:00

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A uniformly charged rod of length L and total charge Q lies along the x axis as shown. Find the magnitudes of the components of the electric field at the point P on the y axis a distance d from the origin.

Answer	$E_x = k_e\lambda \int_0^L \frac{x\,dx}{(d^2+x^2)^{3/2}} = k_e\lambda\left[\frac{1}{d} - \frac{1}{(d^2+L^2)^{1/2}}\right]$
	$E_y = k_e\lambda d \int_0^L \frac{dx}{(d^2+x^2)^{3/2}} = \frac{k_e\lambda L}{d}\frac{1}{(d^2+L^2)^{1/2}}$
When $d \gg L$	$E_x = 0$
	$E_y \cong \frac{k_e Q}{d^2}$



The electric field at point P due to each element of length dx is

$$dE = \frac{k\,dq}{x^2+d^2}$$

And is directed along the line joining the element to point P. The charge element

$$dq = \lambda\,dx$$

The x and y components are

$$dE_x = dE\sin\theta$$

$$dE_y = dE\cos\theta$$

Where:

$$\sin\theta = \frac{x}{\sqrt{x^2+d^2}}$$

$$\cos\theta = \frac{d}{\sqrt{x^2+d^2}}$$

Therefore:

$$E_x = k\lambda \int_0^L \frac{x\,dx}{(x^2+d^2)^{3/2}}$$

$$E_y = k\lambda y \int_0^L \frac{dx}{(x^2 + d^2)^{3/2}}$$

Using the integrals:

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 + a^2}}$$

Evaluate the integral:

$$E_x = k\lambda\left(\frac{1}{d} - \frac{1}{\sqrt{L^2 + d^2}}\right)$$

$$E_y = \frac{k\lambda L}{d}\frac{1}{\sqrt{L^2 + d^2}}$$

Charged rod 3

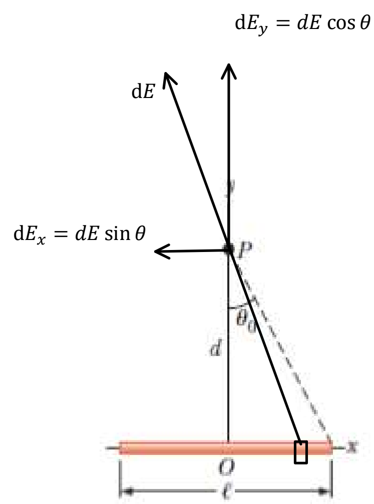
Friday, 29 January, 2021 20:01

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A thin rod of length L and uniform charge per unit length λ lies along the x axis as shown. Find the electric field at P, a distance d from the rod along its perpendicular bisector.

Answer	
By symmetry	$E_x = 0$
	$E_y = 2k_e\lambda d \int_0^{L/2} \frac{dx}{(d^2+x^2)^{3/2}}$
	$E_y = \frac{2k_e\lambda}{d} sin\theta_\circ$
For an infinite length: $\theta_o = 90^\circ$	$E_y = \frac{2k_e\lambda}{d}$
When $d \gg L$	$E_y = \frac{k_e Q}{d^2}$



The electric field at point P, due to each element of length dx, is

$$dE = \frac{k\,dq}{x^2+d^2}$$

and is directed along the line joining the element to point P. By symmetry,

$$E_x = \int dE_x = 0$$

And since:

$$dq = \lambda\,dx$$

$$E = E_y = \int dE_y = \int dE\cos\theta$$

Where: $\cos\theta = \frac{d}{\sqrt{x^2+d^2}}$

Therefore:

$$E = 2E_y = \int 2\,dE\cos\theta = 2k\int \frac{dq}{x^2+d^2}\cos\theta$$

$$E = 2k\lambda\,d\int_0^{L/2} \frac{dx}{(x^2+d^2)^{3/2}}$$

$$\int \frac{\mathrm{d}x}{(x^2 + a^2)^{3/2}} = \frac{x}{a\sqrt{x^2 + a^2}}$$

$$E = \frac{2k\lambda}{d} \frac{\frac{1}{2}L}{\sqrt{\left(\frac{1}{2}L\right)^2 + d^2}}$$

With: $\sin\theta = \frac{\frac{1}{2}L}{\sqrt{\left(\frac{1}{2}L\right)^2 + d^2}}$

$$E = \frac{2k\lambda}{d} \sin\theta$$

For a bar of infinite length, $\theta = 90^{\circ}$

$$E = \frac{2k\lambda}{d}$$

Charged Circular arc

Friday, 29 January, 2021 20:01

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
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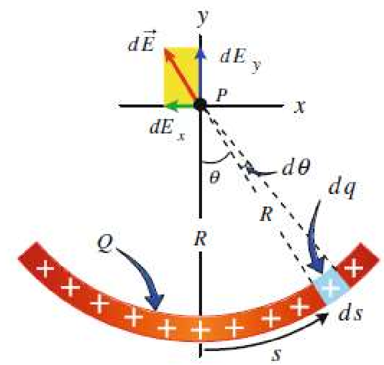
Assume that a rod has a uniformly distributed total positive charge Q. Also assume that the rod is bent into a circular section of radius R and central angle $\Phi\, rad$. Calculate the electric field at the center P of this arc.

Answer	
By symmetry	$E_x = \int dE_x = 0$
	$E_y = \frac{kQ}{R^2\Phi} \int_{-\Phi/2}^{\Phi/2} \cos\theta\, d\theta$
	$E_y = \frac{2kQ}{R^2\Phi} \sin\frac{\Phi}{2}$
With $\lambda = \frac{Q}{R\Phi}$	$E_y = \frac{2k\lambda}{R} \sin\frac{\Phi}{2}$
For a point charge, $\Phi \to 0$; $\left[\lim_{\Phi\to 0} \frac{\sin(\Phi/2)}{(\Phi/2)} = 1\right]$	$E_y = \frac{kQ}{R^2}$
For half a circle, $\Phi = \pi$; $\left[\frac{\sin(\pi/2)}{(\pi/2)} = \frac{2}{\pi}\right]$	$E_y = \frac{2kQ}{\pi R^2} = \frac{2k\lambda}{R}$
For a ring of radius R, $\Phi = 2\pi$	$E_y = 0$



To find the electric field at the center P of this arc, we place coordinate axes such that the axis of symmetry of the arc lies along the $y - axis$ and the origin is at the arc's center. If we let λ represent the linear charge density of this arc which has a length $s = R\Phi$, then:

$$\lambda = \frac{Q}{R\Phi}$$

For an arc element ds subtending an angle $d\theta$ at P, we have:

$$ds = R\, d\theta$$

Therefore, the charge dq on this arc element will be given by:

$$dq = \lambda\, ds = \frac{Q}{R\Phi} R\, d\theta = \frac{Q}{\Phi} d\theta$$

To find the electric field at point P, we first calculate the magnitude of the electric field dE at P due to this element of charge dq, as follows:

$$dE = \frac{k\, dq}{R^2} = \frac{kQ}{R^2\Phi} d\theta$$

The x-component created at P by any charge element dq is canceled by a symmetric charge element on the opposite side of the arc:

$$E_x = \int \mathrm{d}E_x = 0$$

Thus, the perpendicular components of all of the charge elements sum to zero.

The vertical component will take the form:

$$E_y = \int \mathrm{d}E \cos\theta$$

$$E_y = \int \frac{kQ}{R^2\Phi} \cos\theta \, \mathrm{d}\theta$$

$$E_y = \frac{kQ}{R^2\Phi} \int_{-\Phi/2}^{\Phi/2} \cos\theta \, \mathrm{d}\theta$$

$$E_y = \frac{kQ}{R^2\Phi} \; sin\theta \Big|_{-\Phi/2}^{\Phi/2}$$

$$E_y = \frac{kQ}{R^2\Phi}\left[\sin\frac{\Phi}{2} - \sin\left(-\frac{\Phi}{2}\right)\right]$$

$$E_y = \frac{kQ}{R^2\Phi}\left[\sin\frac{\Phi}{2} + \sin\left(\frac{\Phi}{2}\right)\right]$$

$$E_y = \frac{2kQ}{R^2\Phi}\sin\frac{\Phi}{2}$$

$$E_y = \frac{2kQ}{R^2\Phi}\sin\frac{\Phi}{2}$$

$$E_y = \frac{2k\lambda}{R}\sin\frac{\Phi}{2}$$