Chapter 28

1

Direct Current Circuits

1

Lecture 03

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RC Circuits

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RC Circuits

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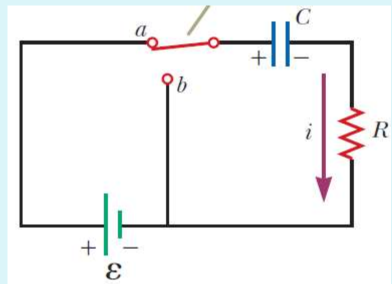
In direct current circuits containing capacitors, the current may vary with time.

The current is still in the same direction.

An RC circuit will contain a series combination of a resistor and a capacitor.



3

Charging a Capacitor

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- When the circuit is completed, the capacitor starts to charge.
- At the instant the switch is closed, the charge on the capacitor is zero.
- The capacitor continues to charge until it reaches its maximum charge ($Q = C\varepsilon$).
- Once the capacitor is fully charged, the current in the circuit is zero.
- As the plates are being charged, the potential difference across the capacitor increases, such that:

$$\varepsilon = V_C + V_R$$

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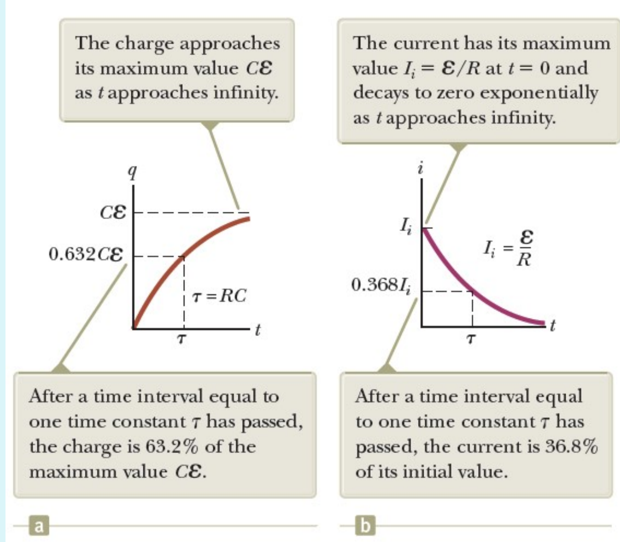
Charging a Capacitor



5

Time Constant

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τ is the *time constant*

$$\tau = RC$$

The time constant represents the time required for the charge to increase from zero to 63.2% of its maximum.

τ has units of time (s)

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Charging a Capacitor, Equations

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When the circuit is completed, the capacitor starts to charge. At the instant the switch is closed ($\boldsymbol{t = 0\ s}$)	
The **charge** on the **capacitor** is **zero**:	• $Q_C = 0$
The **potential** difference across the **capacitor** is **zero**:	• $V_C = 0$
The **potential** difference across the **resistor** is **maximum**:	• $V_R = \varepsilon$
The **current** in the circuit is **maximum**:	• $I_{max.} = \frac{\varepsilon}{R}$

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Charging a Capacitor , Equations

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After a very long time ($\boldsymbol{t} \rightarrow \infty$),	
the **charge** on the **capacitor** is **maximum**:	• $Q_{max.} = C\ \varepsilon$
The **potential** difference across the **capacitor** is **maximum**:	• $V_C = \varepsilon$
The **potential** difference across the **resistor** is **zero**:	• $V_R = 0$
The **current** in the circuit is **zero**:	• $I = 0$
The **energy** stored in the capacitor is **maximum**:	• $U_{max.} = \frac{1}{2}Q\ \varepsilon = \frac{1}{2}C\ \varepsilon^2$

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Charging a Capacitor , Equations

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While charging the capacitor ($0 < t < \infty$),

The **charge** on the **capacitor increases** with time:

- $Q(t) = Q_{max.}\,(1 - e^{-\frac{t}{\tau}})$

The **potential** difference across the **capacitor increases** with time:

- $V_C(t) = \varepsilon\,(1 - e^{-\frac{t}{\tau}})$

The **energy** stored in the **capacitor increases** with time:

- $U(t) = U_{max.}\,(1 - e^{-\frac{2t}{\tau}})$

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Charging a Capacitor , Equations

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The **potential** difference across the **resistor decreases** with time:

- $V_R(t) = \varepsilon\, e^{-\frac{t}{\tau}}$

The **current** in the circuit **decreases** with time:

- $I(t) = I_{max.}\, e^{-\frac{t}{\tau}}$

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Discharging a Capacitor

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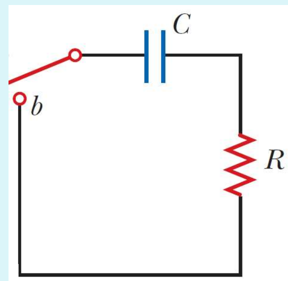
- Imagine that the capacitor in the figure shown is completely charged.
- The initial potential difference across the capacitor is:

$$V_C = \frac{Q_i}{C}$$

- There is zero potential difference across the resistor because $i = 0$.

$$V_R = 0$$

$$I = 0$$



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Discharging a Capacitor

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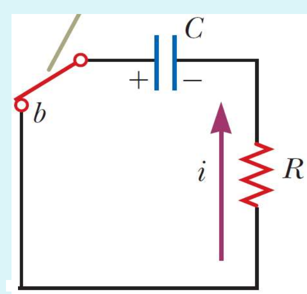
- When the switch is closed, charge begins to flow through resistor R from the right side of the capacitor toward the left side, until the capacitor is fully discharged.
- The voltage across the resistor at any instant **equals** that across the capacitor.
- At some time t during the discharge, the current in the circuit is $I(t)$ and the charge on the capacitor is $Q(t)$.
- As the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged.



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Discharging a Capacitor

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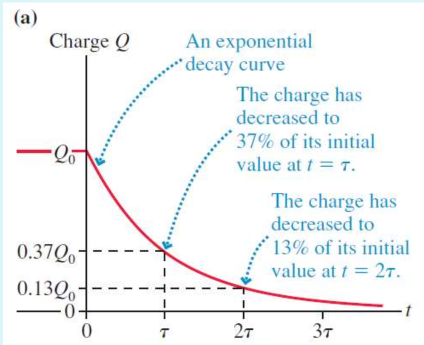

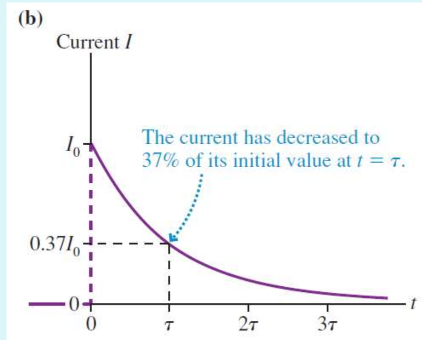

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Discharging a Capacitor , Equations

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When the capacitor starts to discharge. At the instant the switch is closed ($t = 0\ s$)

The **charge** on the **capacitor** is **maximum**:	• $Q_C = Q_{max.}$
The **potential** difference between the plates of the **capacitor** is **maximum**:	• $V_C = \frac{Q_{max.}}{C}$
The **potential** difference between the ends of the **resistor** is **maximum**:	• $V_R = V_C = \frac{Q_{max.}}{C}$
The **current** in the circuit is **maximum**:	• $I_{max.} = \frac{V_R}{R} = \frac{Q_{max.}}{RC}$
The **energy** stored in the capacitor is **maximum**:	• $U_{max.} = \frac{Q^2}{2C}$

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Discharging a Capacitor , Equations

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After a very long time ($t \to \infty$),

The **charge** on the **capacitor** is **zero** :	• $Q = 0$
The **potential** difference between the plates of the **capacitor** is **zero** :	• $V_C = 0$
The **potential** difference between the ends of the **resistor** is **zero**:	• $V_R = 0$
The **current** in the circuit is **zero**:	• $I = 0$
The **energy** stored in the capacitor is **zero** :	• $U = 0$

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Discharging a Capacitor , Equations

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While discharging the capacitor ($0 < t < \infty$),

The **charge** on the **capacitor decreases** with time:	• $q(t) = Q_{max.}\, e^{-\frac{t}{\tau}}$
The **potential** difference between the plates of the **capacitor decreases** with time:	• $V_C(t) = \frac{Q_{max.}}{C}\, e^{-\frac{t}{\tau}}$
The **energy** stored in the **capacitor decreases** with time:	• $U(t) = \frac{Q^2}{2C}\, e^{-\frac{2t}{\tau}}$

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Discharging a Capacitor , Equations

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The **potential** difference between the ends of the **resistor decreases** with time:

- $V_R(t) = \frac{Q_{max.}}{C} e^{-\frac{t}{\tau}}$

The **current** in the circuit **decreases** with time:

- $I(t) = \frac{Q_{max.}}{RC} e^{-\frac{t}{\tau}}$

The **energy** delivered to the **resistor increases** with time:

- $U(t) = \frac{Q^2}{2C}\left(1 - e^{-\frac{2t}{\tau}}\right)$

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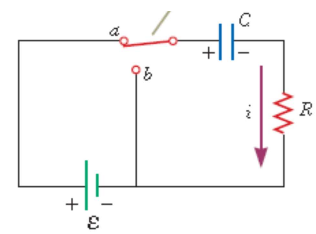
Charging a Capacitor in an RC Circuit

Tuesday, 2 February, 2021 21:45

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

An uncharged capacitor and a resistor are connected in series to a battery as shown, where $\mathcal{E} = 12.0\ V$, $C = 5.00\ \mu F$, and $R = 8.00 \times 10^5\ \Omega$. The switch is thrown to position a. Find:



- the time constant of the circuit,
- the maximum charge on the capacitor,
- the maximum current in the circuit,
- the charge as functions of time,
- the current as functions of time,
- What fraction of the final charge Q is on the capacitor at $t = 8\ s$?
- What fraction of the initial current I is still flowing at $t = 8\ s$?

SOLUTION

Evaluate the time constant of the circuit:

$$\tau = RC = (8.00 \times 10^5 \Omega)(5.00 \times 10^{-6} F) = 4.00s$$

Evaluate the maximum charge on the capacitor:

$$Q = C\mathcal{E} = (5.00\mu F)(12.0V) = 60.0\mu C$$

Evaluate the maximum current in the circuit:

$$I_i = \frac{\mathcal{E}}{R} = \frac{12.0V}{8.00 \times 10^5 \Omega} = 15.0\mu A$$

Use these values to find the charge and current as functions of time:

$$(1)\quad q(t) = 60.0(1 - e^{-t/4.00})$$

$$(2)\quad I(t) = 15.0e^{-t/4.00}$$

q is in microcoulombs, I is in microamperes, and t is in seconds.

The fraction of the final charge on the capacitor is:

$$\frac{q}{Q} = 1 - \mathrm{e}^{-t/RC} = 1 - \mathrm{e}^{-8/4} = 0.86$$

The fraction of the initial current still flowing is:

$$\frac{I}{I_0} = \mathrm{e}^{-t/Rc} = \mathrm{e}^{-8/4} = 0.14$$

Discharging a Capacitor in an RC Circuit

Tuesday, 2 February, 2021 21:50

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

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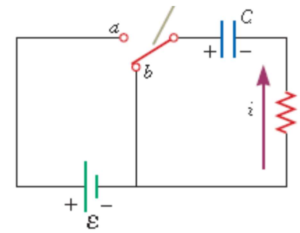
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Consider a capacitor of capacitance C that is being discharged through a resistor of resistance R as shown.



- After how many time constants is the charge on the capacitor one-fourth its initial value? and
- What is the current at this time?
- The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth its initial value?

SOLUTION

Substitute $q(t) = Q/4$ into $(q(t) = Q_\circ e^{-t/RC})$:

$$\frac{Q}{4} = Qe^{-t/RC}$$

$$\frac{1}{4} = e^{-t/RC}$$

Take the logarithm of both sides of the equation and solve for t:

$$-1n4 = -\frac{t}{RC}$$

$$t = RC1n4 = 1.39RC = 1.39\tau$$

The current at $t = 1.39\tau$ is:

$$I = I_0 e^{-t/\tau} = I_0 e^{-1 \cdot 39} = 0.25 I_0$$

SOLUTION

express the energy stored in the capacitor at any time t:

$$(1)\ \ U(t) = \frac{q^2}{2C} = \frac{Q^2}{2C} e^{-2t/RC}$$

Substitute $U(t) = \frac{1}{4}(Q^2/2C)$ into Equation (1):

$$\frac{1}{4}\frac{Q^2}{2C} = \frac{Q^2}{2C} e^{-2t/RC}$$

$$\frac{1}{4} = e^{-2t/RC}$$

Take the logarithm of both sides of the equation and solve for t:

$$-1n4 = -\frac{2t}{RC}$$

$$t = \frac{1}{2}RC1n4 = 0.693RC = 0.693\tau$$

Notice that because the energy depends on the square of the charge, the energy in the capacitor drops more rapidly than the charge on the capacitor.

What if you want to describe the circuit in terms of the time interval required for the charge to fall to one half its original value rather than by the time constant τ?

In one half-life, the charge falls from Q to $Q/2$. Therefore,

$$\frac{Q}{2} = Qe^{-t_{1/2}/RC} \rightarrow \frac{1}{2} = e^{-t_{1/2}/RC}$$

which leads to

$$t_{1/2} = 0.693\tau$$