

Thermodynamics: An Engineering Approach, 6th Edition
Yunus A. Cengel, Michael A. Boles
McGraw-Hill, 2008

Chapter 7

ENTROPY

ENTROPY

In General, Clausius Inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

The Equality in the Clausius inequality holds for Totally or Just Internally Reversible Cycles and the Inequality for the Irreversible ones.

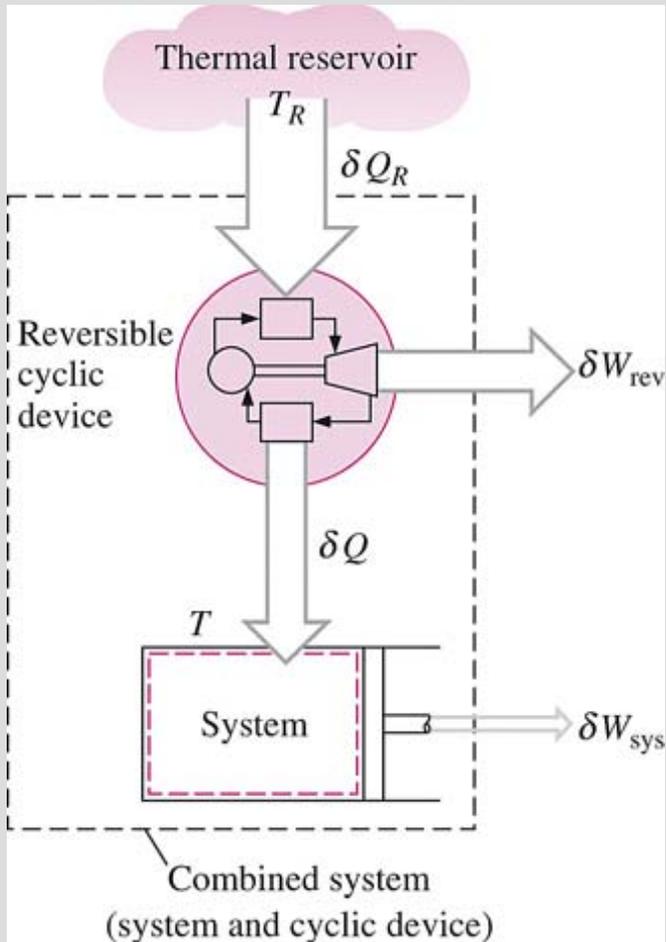
Clausius Inequality
(Irreversible)

$$\oint \frac{\delta Q}{T} < 0$$

Clausius Equality
(Internally, reversible)

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

ENTROPY



Applying Energy equation to combined system

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}$$

$$\delta W_C = \delta Q_R - dE_C$$

Where $\delta W_C = \delta W_S + \delta W_{rev}$

For a reversible cyclic device $\left(\frac{Q_H}{Q_L} \right)_{rev} = \frac{T_H}{T_L}$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T}$$

$$\delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

If we let the combined system undergoes a cycle

Then $W_C = T_R \oint \frac{\delta Q}{T}$ $\oint \frac{\delta Q}{T} \leq 0$

Denoted by (C)

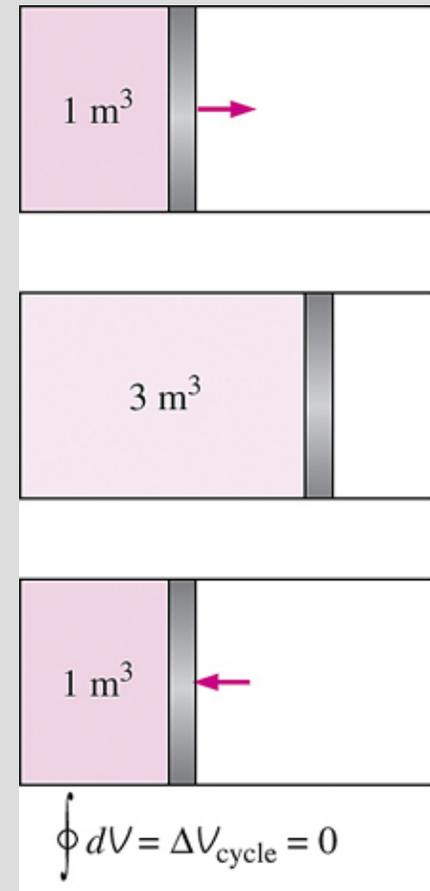
DEFINITION OF ENTROPY

A NEW PROPERTY CALLED ENTROPY

(S) AND IS GIVEN BY DEFINITION AS:

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$



The net change in volume (a property) during a cycle is always zero.

A Special Case: Internally Reversible Isothermal Heat Transfer Processes

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0} \right)_{\text{int rev}} = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}}$$

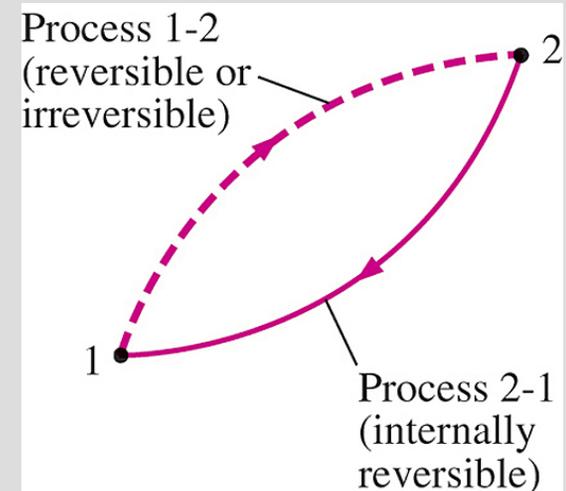
$$\Delta S = \frac{Q}{T_0}$$

This equation is particularly useful for determining the entropy changes of Thermal Energy Reservoirs.

THE INCREASE OF ENTROPY PRINCIPLE

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$



A cycle composed of a reversible and an irreversible process.

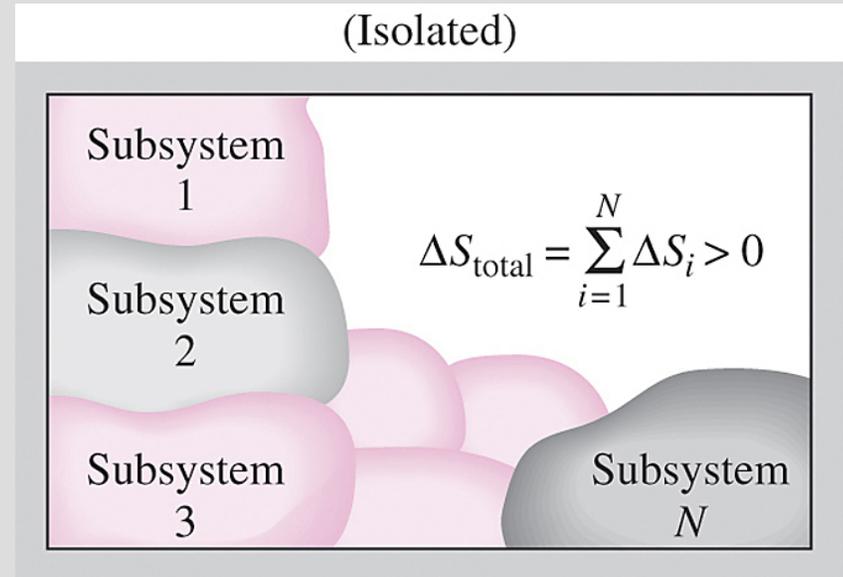
Some entropy is generated or created during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation S_{gen} is always a positive quantity or zero.

THE INCREASE OF ENTROPY PRINCIPLE

The entropy change of an isolated system is the sum of the entropy changes of its components, and **is never less than zero.**

$$\Delta S_{\text{isolated}} \geq 0$$

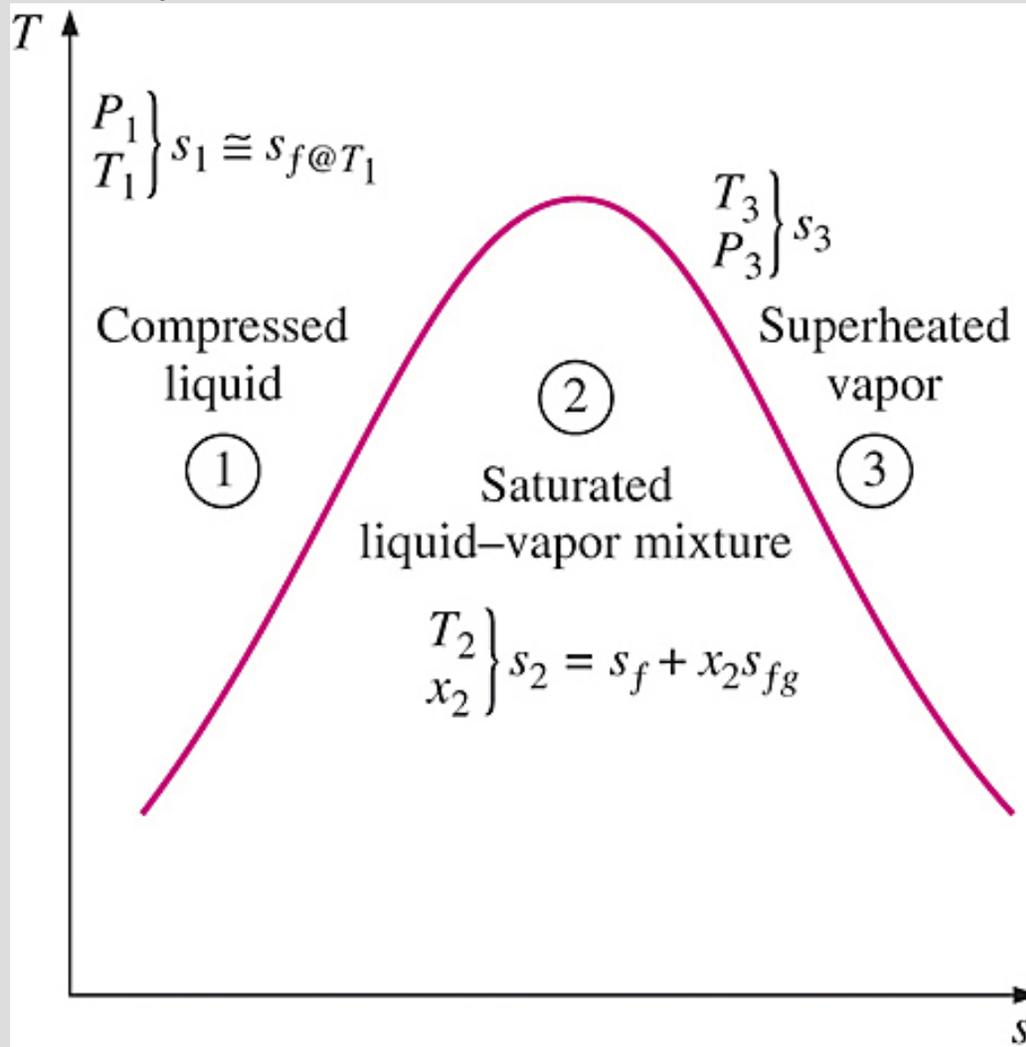


$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

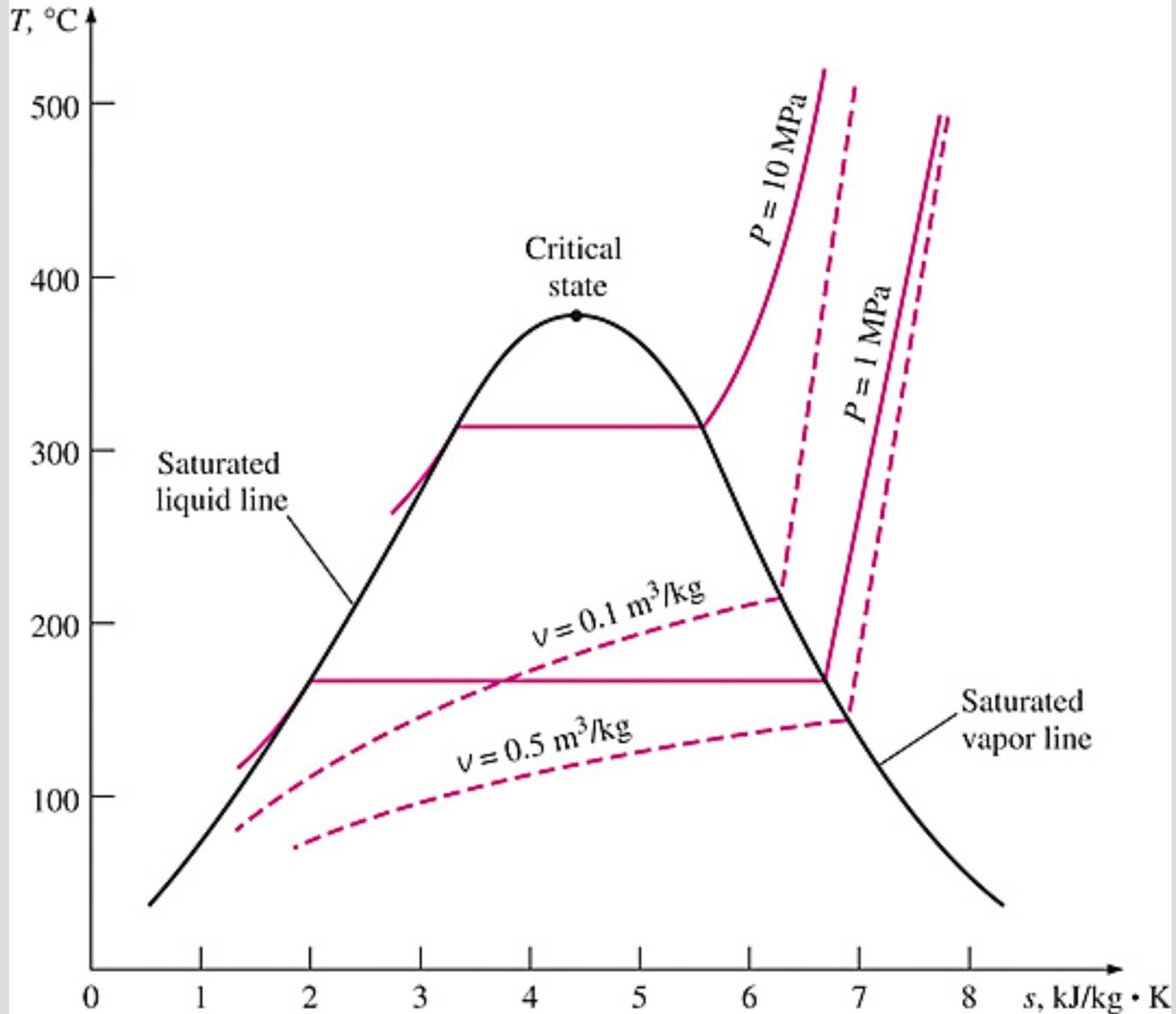
$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.



The entropy of a pure substance is determined from the tables (like other properties).



Schematic of the (T-S) diagram for water.

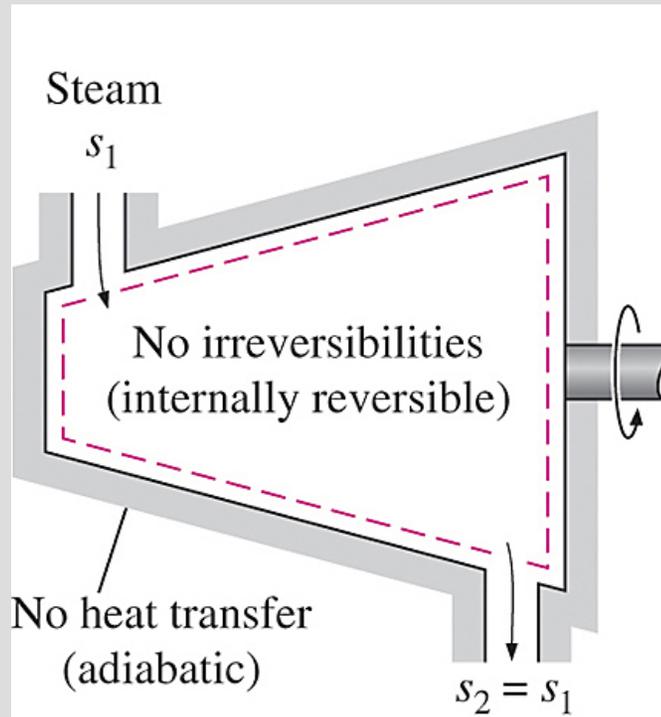
Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \quad (\text{kJ/K})$$

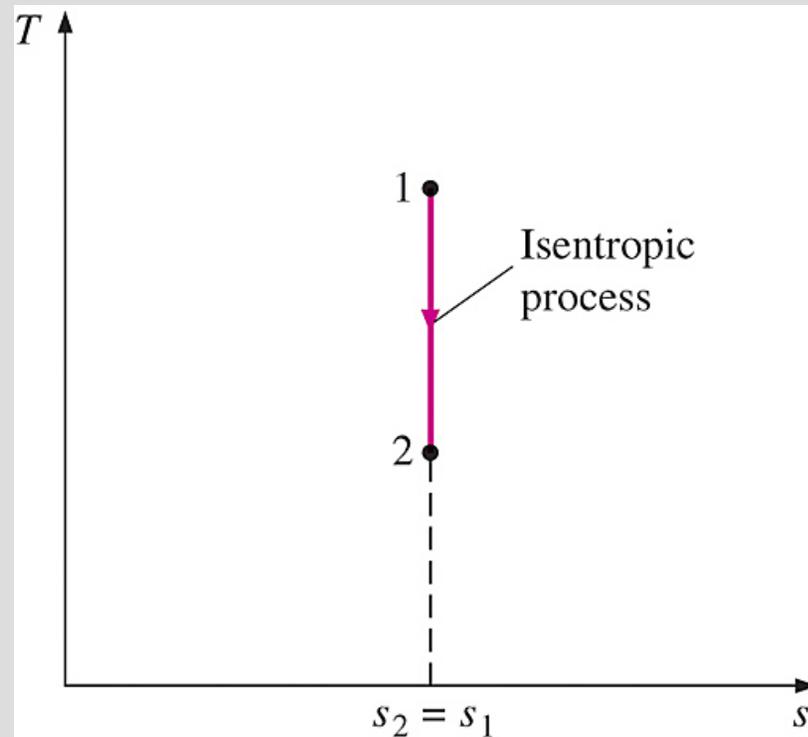
ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **Isentropic process**.

$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1 \quad (\text{kJ/kg} \cdot \text{K})$$

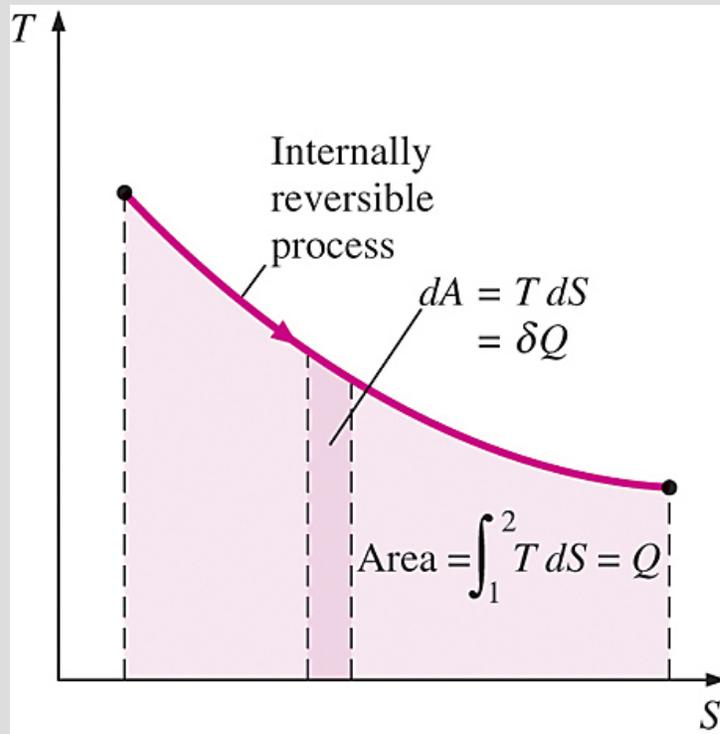


During an internally **reversible, adiabatic** (isentropic) process, the entropy remains constant.



The isentropic process appears as a *vertical* line segment on a T - S diagram.

PROPERTY DIAGRAMS INVOLVING ENTROPY



On a T - S diagram, the area under the process curve represents the heat transfer for internally reversible processes.

$$\delta Q_{\text{int rev}} = T dS$$

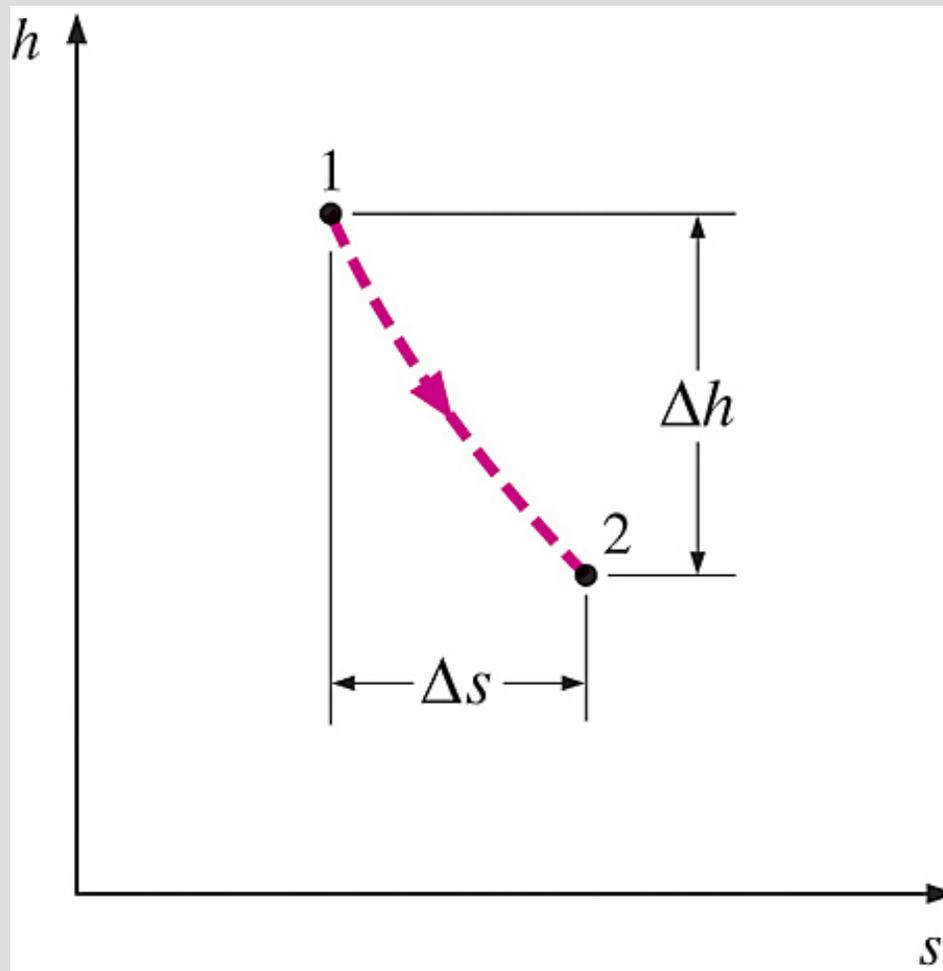
$$Q_{\text{int rev}} = \int_1^2 T dS$$

$$\delta q_{\text{int rev}} = T ds$$

$$q_{\text{int rev}} = \int_1^2 T ds$$

$$Q_{\text{int rev}} = T_0 \Delta S$$

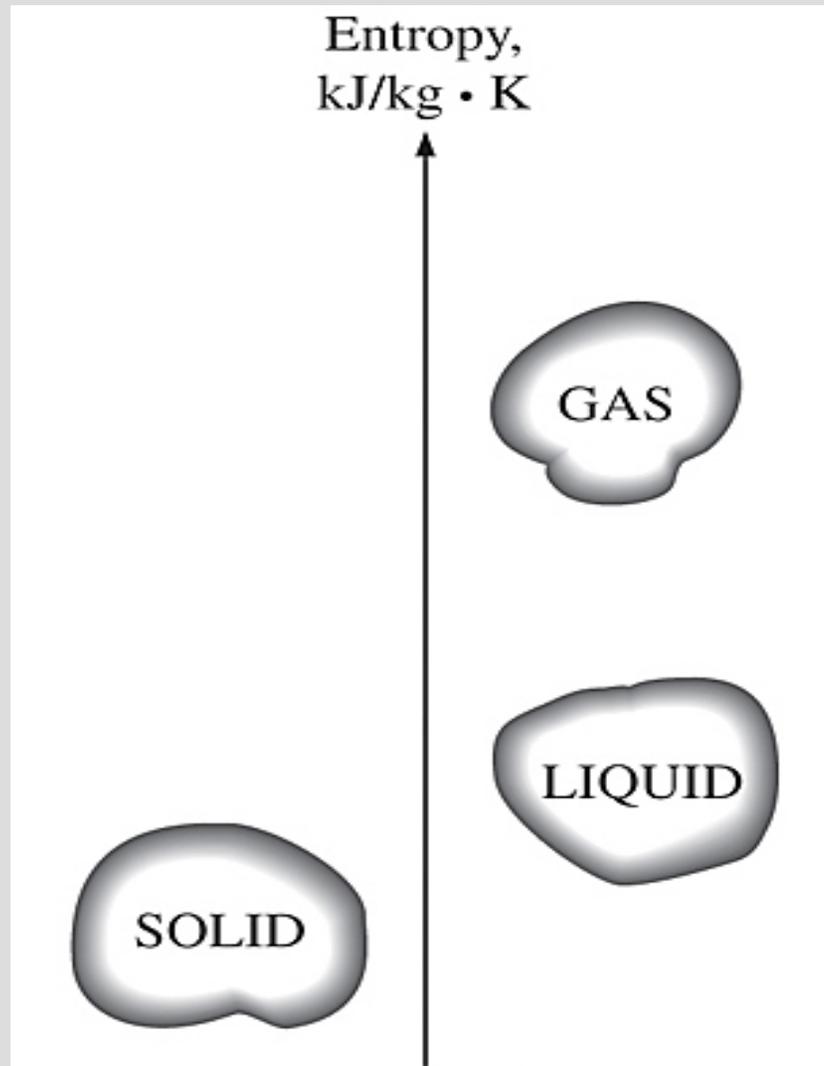
$$q_{\text{int rev}} = T_0 \Delta s$$



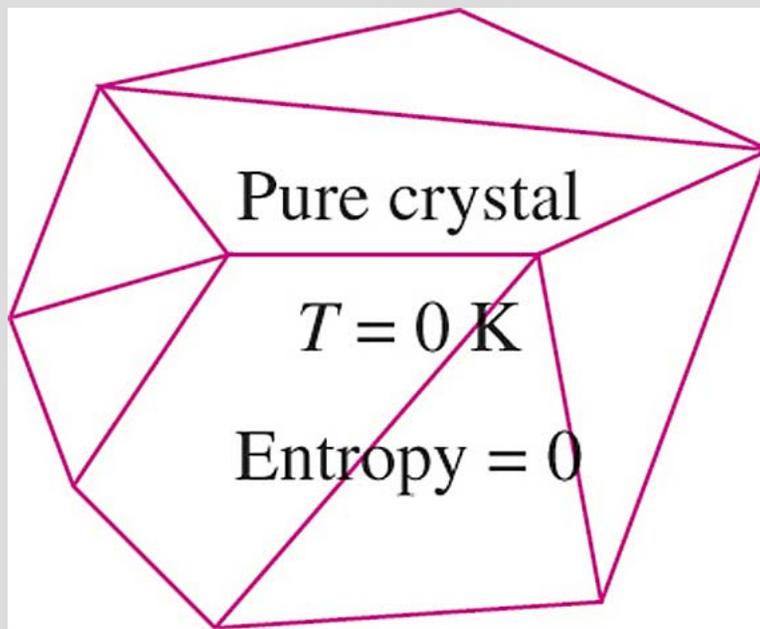
For adiabatic steady-flow devices, the vertical distance Δh on an h - s diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

The h - s diagram is called Mollier diagram

WHAT IS ENTROPY?



The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.



A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (the third law of thermodynamics).

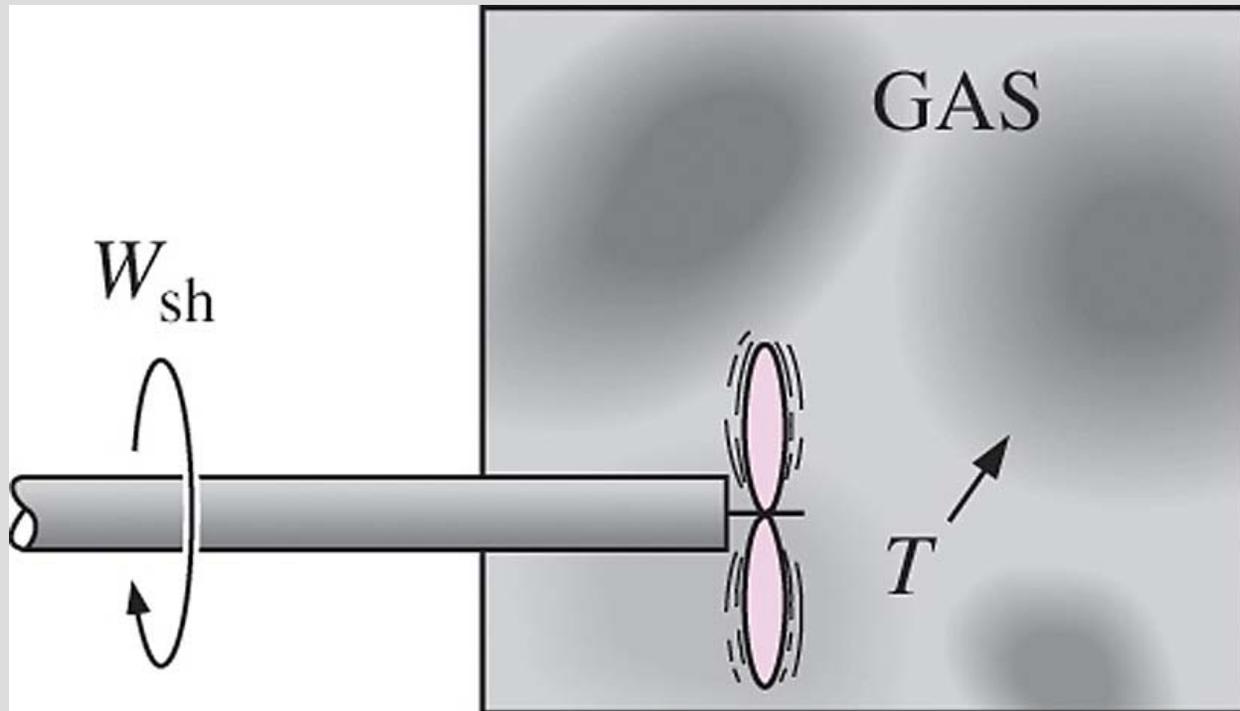
Boltzmann relation

$$S = k \ln p \quad k = 1.3806 \times 10^{-23} \text{ J/K}$$

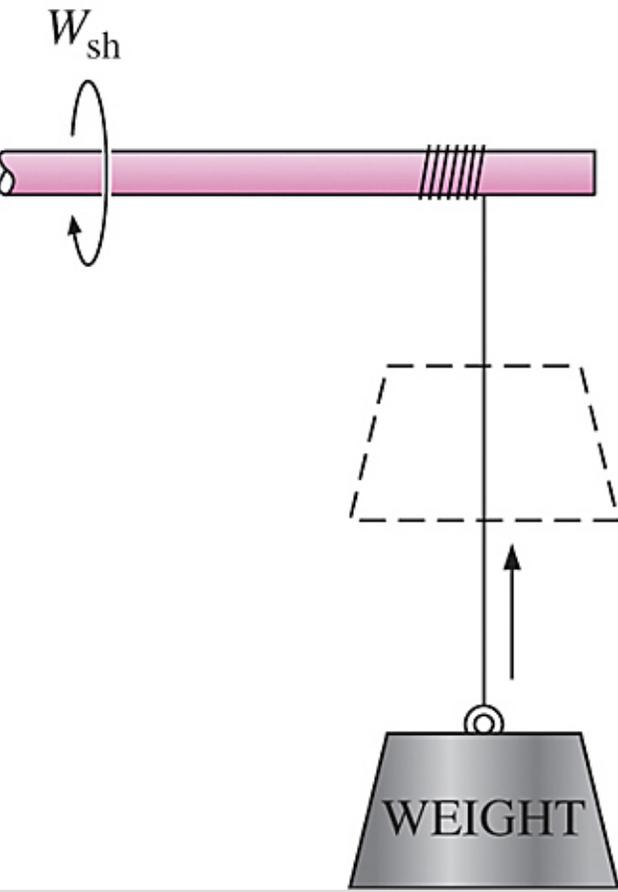
(P) is called Thermodynamic Probability



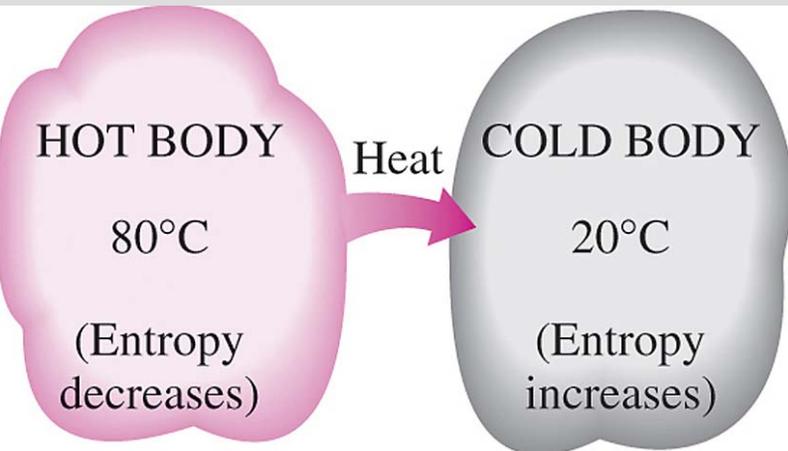
Disorganized energy does not create much useful effect, no matter how large it is.



The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

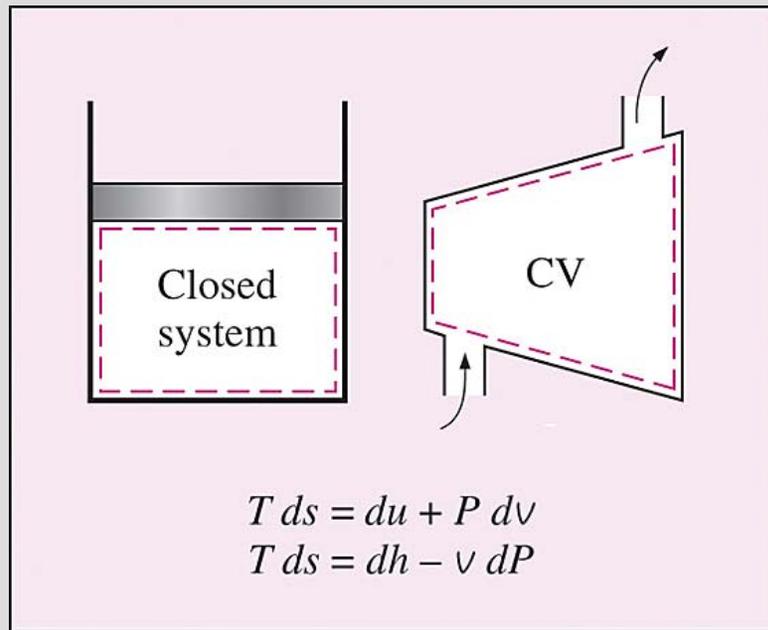


In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.



During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

THE $T - dS$ RELATIONS



The $T dS$ relations are valid for both reversible and irreversible processes and for both closed and open systems.

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

$$\delta Q_{\text{int rev}} = T dS$$

$$\delta W_{\text{int rev,out}} = P dV$$

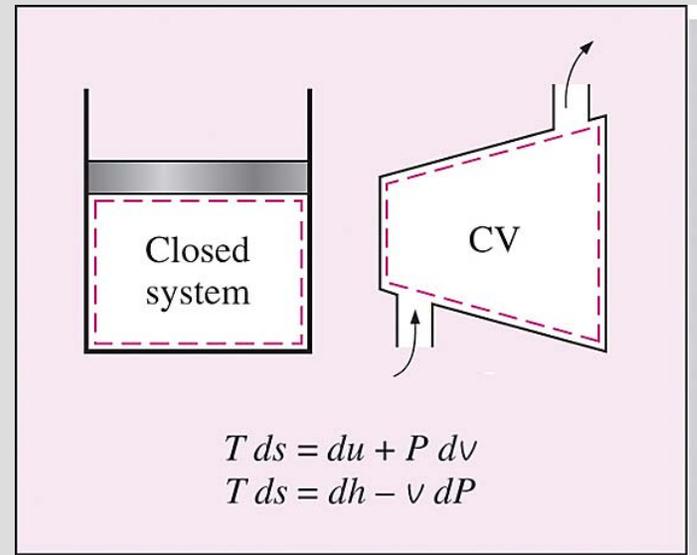
$$T dS = dU + P dV \quad (\text{kJ})$$

$$T ds = du + P dv \quad (\text{kJ/kg})$$

The first $T - dS$, or Gibbs equation

THE $T - dS$ RELATIONS

The $T dS$ relations are valid for both reversible and irreversible processes and for both closed and open systems.



$$h = u + Pv$$

$$\left. \begin{array}{l} dh = du + P dv + v dP \\ T ds = du + P dv \end{array} \right\} T ds = dh - v dP$$

The second $T - dS$ equation

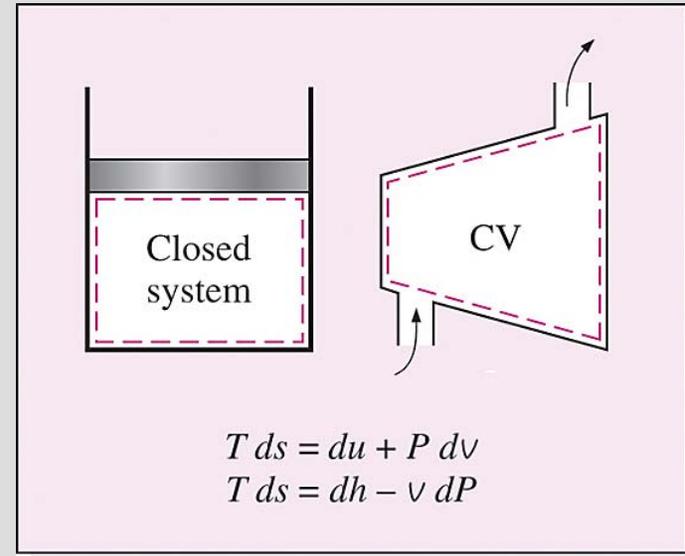
THE $T ds$ RELATIONS

Closed System

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

Open System

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$



The $T ds$ relations are valid for both reversible and irreversible processes and for both closed and open systems.

Differential changes in entropy in terms of other properties

ENTROPY CHANGE OF LIQUIDS AND SOLIDS

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

Since $dv \cong 0$ for liquids and solids

$$ds = \frac{du}{T} = \frac{c dT}{T}$$

since $c_p = c_v = c$ and $du = c dT$

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

$$\text{Liquids, solids:} \quad s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{\text{avg}} \ln \frac{T_2}{T_1} \quad (\text{kJ/kg} \cdot \text{K})$$

For **Isentropic Process** of an incompressible substance

$$\text{Isentropic:} \quad s_2 - s_1 = c_{\text{avg}} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

THE ENTROPY CHANGE OF IDEAL GASES

From the first $T ds$ relation

$$ds = \frac{du}{T} + \frac{P dv}{T}$$
$$du = c_v dT$$
$$P = RT/v$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

From the second $T ds$ relation

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$
$$dh = c_p dT$$
$$v = RT/P$$

$$ds = C_p \frac{dT}{T} - R \ln \frac{dP}{P}$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$Pv = RT$$
$$du = C_v dT$$
$$dh = C_p dT$$

A broadcast
from channel IG.

Constant Specific Heats (Approximate Analysis)

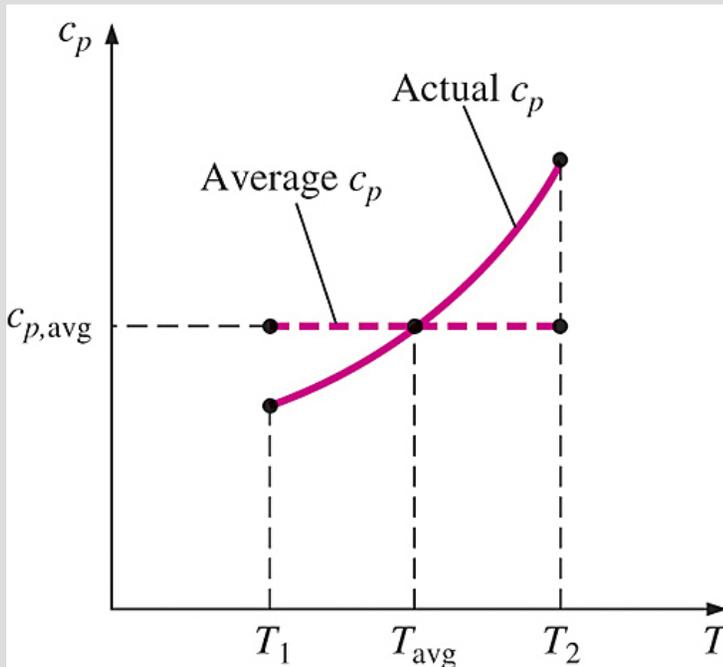
$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$s_2 - s_1 = c_{p,avg} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

(kJ/kg · K)



Entropy change of an ideal gas on a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,avg} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,avg} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function s° as

$$s^\circ = \int_0^T c_p(T) \frac{dT}{T}$$

$$\int_1^2 c_p(T) \frac{dT}{T} = s_2^\circ - s_1^\circ$$

On a unit-mass basis

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kg} \cdot \text{K})$$

On a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

T, K	$s^\circ, \text{kJ/kg} \cdot \text{K}$
·	·
·	·
·	·
300	1.70203
310	1.73498
320	1.76690
·	·
·	·
·	·
(Table A-17)	

The entropy of an ideal gas depends on both T and P . The function s represents only the temperature-dependent part of entropy.

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2} \right)^{R/c_v}$$

$$R = c_p - c_v, k = c_p/c_v$$

$$\text{and thus } R/c_v = k - 1$$

$$\left(\frac{T_2}{T_1} \right)_{s = \text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

VALID FOR

- *ideal gas
- *isentropic process
- *constant specific heats

The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{V_1}{V_2}\right)^k$$

$$TV^{k-1} = \text{constant}$$

$$TP^{(1-k)/k} = \text{constant}$$

$$PV^k = \text{constant}$$

Isentropic Processes of Ideal Gases

Variable Specific Heats (Exact Analysis)

$$0 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \longrightarrow s_2^\circ = s_1^\circ + R \ln \frac{P_2}{P_1}$$

Relative Pressure and Relative Specific Volume

$$\frac{P_2}{P_1} = \exp \frac{s_2^\circ - s_1^\circ}{R}$$

$\exp(s^\circ/R)$ is the relative pressure P_r

$$\frac{P_2}{P_1} = \frac{\exp(s_2^\circ / R)}{\exp(s_1^\circ / R)}$$

The use of P_r data for calculating the final temperature during an isentropic process.

The use of v_r data for calculating the final temperature during an isentropic process

Process: isentropic

Given: P_1 , T_1 , and P_2

Find: T_2

T	P_r
⋮	⋮
⋮	⋮
⋮	⋮
T_2	← read $P_{r2} = \frac{P_2}{P_1} P_{r1}$
⋮	⋮
⋮	⋮
T_1	→ read P_{r1}
⋮	⋮
⋮	⋮

Process: isentropic

Given: v_1 , T_1 , and v_2

Find: T_2

T	v_r
⋮	⋮
⋮	⋮
⋮	⋮
T_2	← read $v_{r2} = \frac{v_2}{v_1} v_{r1}$
⋮	⋮
⋮	⋮
T_1	→ read v_{r1}
⋮	⋮
⋮	⋮

Iisentropic Processes of Ideal Gases

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \rightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{T_2}{T_1} \frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$

T/P_r is the relative specific volume v_r .

THE END

